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A TREATISE

ON

MAGNETISM.

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A TREATISE

ON

MAGNETISM

Designed for the use of Students in the University.

BY

GEORGE BIDDELL AIRY, M.A. LL.D. D.C.L.

FORMERLY FELLOW, NOW HONORARY FELLOW, OF TRINITY COLLEGE;
LATE LUCASIAN PROFESSOR OF MATHEMATICS, AFTERWARDS PLUMIAN PROFESSOR
OF ASTRONOMY AND EXPERIMENTAL PHILOSOPHY,
IN THE UNIVERSITY OF CAMBRIDGE;
ASTRONOMER ROYAL.

London:
MACMILLAN AND CO.
1870.

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A/d

IN ENDEAVOURING TO INTRODUCE INTO

THE UNIVERSITY OF CAMBRIDGE

A PHYSICAL SUBJECT, OF MATHEMATICAL CHARACTER,

HITHERTO UNRECOGNIZED

IN ITS ACADEMICAL COURSE;

I VENTURE TO INSCRIBE THIS WORK

TO MY HONOURED FRIEND

SIR JOHN FREDERICK HERSCHEL, BARONET, K.H.,

ONE OF A SMALL BAND

WHO BY THEIR PRIVATE EFFORTS

ESTABLISHED IN THE UNIVERSITY

THE FORM OF MATHEMATICS

THEN AND NOW ACCEPTED IN THE SCIENTIFIC WORLD.

G. B. AIRY.

1870, *November.*

ADVERTISEMENT.

In the spring of 1864 I was honoured with a request from the Vice-Chancellor of the University of Cambridge to deliver the Lecture on Sir Robert Rede's foundation: and in my Address in the Senate House on 1864 May 10, in speaking of the advantages which might be expected to follow the establishment of that Lecture, I took occasion to point out what appeared to be defects in the system of education in the University as connected with Mathematical Physics. I followed up this oral remark by a letter to the Vice-Chancellor; and the subject by degrees attracted the attention of the University.

Remarking that, in addition to excellent works on Spherical and Gravitational Astronomy, General Mechanics, Hydrostatics, Pneumatics, and common Optics, a treatise on Physical Optics existed in the University; it appeared desirable to provide for the subjects of Tides, Waves, Sound, Electricity, and Magnetism: as well as for some of the modifications of

Pure Mathematics specially applicable to the Observing Sciences. The foundations for treatises on Electricity, Tides, and Waves, exist in articles in the Encyclopædia Metropolitana; and I trust that some Resident Member of the University may be induced to exhibit these branches of science in a form adapted to University Education. In redemption of the engagement into which I had virtually entered to place the other subjects before the University, I have published works on Probabilities and on Partial Differential Equations (both with express reference to Mathematical Physics), and on Sound. I now close my part of the undertaking by this Treatise on Magnetism.

I am indebted to James Glaisher, Esq. F.R.S. and F.R.A.S., and to James Carpenter, Esq. F.R.A.S., of the Royal Observatory, for much assistance in the preparation of the diagrams inserted in the pages of this work.

G. B. AIRY.

ROYAL OBSERVATORY, GREENWICH,
1870, *November*.

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Page 73, line 11, *for* 83, *read* 85.
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ON MAGNETISM.

SECTION I.

PHYSICAL EXTENSION OF MAGNETISM, AND LIMITATION OF ITS TREATMENT IN THE PRESENT WORK.

1. *Dissemination of magnetism through many components of the earth, and probable cosmical extension of magnetism.*

In ordinary observation, magnetism is scarcely known except as existing in iron and especially in steel, and as related in some obscure manner to the earth. But there is reason to believe that it is one of the most extensively diffused agents in nature. It can be traced not only in iron but also in every substance into which iron (one of the most widely spread substances in nature) enters into composition. It is found in nickel and other substances, and even in some gases. Wherever a galvanic current exists in nature, whether produced by chemical action or appearing in the thermo-electric form as originating from the effects

of heat at the place of union of different substances, magnetic effects can be elicited. On the larger scale, it is certain that the whole Earth acts as a combination of magnets, and there is reason to think that the Sun and the Moon also act as magnets.

2. *Our accurate knowledge of magnetism is limited to the magnetism of iron, steel, and the Earth, to which this work will be confined: with allusion finally to galvanism and thermo-electricity.*

The laws of magnetic force, however, have been experimentally examined with philosophical accuracy, only in its connexion with iron and steel; and, by inferences bearing considerable probability, in the influences excited by the Earth as a whole. The accurate portions of the following work will therefore be confined to the investigations connected with these metals and the Earth. But it will be advantageous, before terminating the treatise, to allude in a more general way to the laws of the connexion between magnetism on the one hand and galvanism and thermo-electricity on the other hand.

SECTION II.

Properties of Steel Magnets.

3. *The steel magnets will be supposed to be slender bars: usually they will be supposed to be straight.*

As a general rule, it is found impracticable to give magnetism, admitting of careful experimental investigation, to a mass of steel of any form except that of a long bar, straight or bent. (Detached bar-magnets are usually made of uniform breadth throughout: compass-needles, and other mounted magnets, are frequently made with pointed ends, as having smaller moment of inertia in proportion to the energy of their magnetism.) The mathematical investigations which follow will be confined to the case of straight bars, in which the length greatly exceeds the breadth. General reference will however be made to the horse-shoe magnet.

4. *Definition of a steel magnet: the definition sometimes applies to iron bars.*

The practical definition of a magnet is, "a bar of steel which, when so suspended or so mounted on a fine point that it can vibrate freely in the horizontal plane, will take a definite direction; and, if disturbed from

that direction, will return to it by a series of vibrations, gradually diminishing in extent, from the effect of atmospheric resistance, &c." As a general rule, the material possessing this property to a degree admitting of experimental examination must be steel. In some exceptional cases however (to be hereafter mentioned) the same properties may be given in a minor degree to bars of iron.

5. *Size of magnets most convenient for experiments.*

For a few important experiments, to be mentioned below, it is desirable to be provided with a large magnet, perhaps one foot or two feet in length. But, generally, the best magnets for experiment are small compass-needles, mounted and unmounted. These are capable of possessing a great magnetic power in proportion to their weight, and they can be procured at small expense.

6. *Mounting of magnets for experiments.*

In experiments where the position taken by the magnet, or its vibration, or its displacement by the action of an external magnetic substance, is to be observed, it is desirable that the magnet (and, if suspended, its suspending apparatus) should be inclosed in a glass case. For many ordinary experiments, the support of the magnet upon a fine point, as in the common compass, is sufficiently delicate; especially if the point be made of the hard iridium-ore, now universally employed for the compasses of the Royal Navy. But for

delicate experiments suspension is far superior. A very small magnet may be carried by a single fibre as spun by the silk-worm: a larger magnet may be supported by a manufacturer's silk thread, formed by the union of six or seven of the silk-worm's threads: and the largest may be suspended by a skein consisting of a number of these threads in parallel lines. In all these cases of suspension, the torsion-power of the support is very small, and there is an almost total absence of friction properly so called.

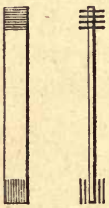
7. *The opposite ends of a magnet have different properties. Explanation of the terms "red" and "blue" magnetism, and of the symbols employed to represent them. Allusion to horse-shoe magnets.*

Understanding then that one end of the magnet thus freely suspended will point to the direction called "Magnetic North" (not generally coinciding with Astronomical or Geographical North, but at the present time, in Greenwich, about 20° west of North), and that the other end will point to the "Magnetic South," and that if the magnet be disturbed in direction it immediately returns to its first position, it is evident that the opposite ends of the magnet possess different properties. The magnetism of the end of the magnet which points nearly to the geographical north will be called *red* magnetism, and that of the opposite end will be called *blue* magnetism. The student is particularly requested to observe that these words "red" and "blue" have here no meaning whatever, except as distinguishing the two

ends. (The words, from their brevity, and their applicability to the colour of the paint put on magnets, are convenient: it has long been the custom of tradesmen to paint with red the north-seeking end of a magnet.) In the diagrams, the red end will be distinguished by a cross-hatching and the blue end by a longitudinal hatching. (It is usual for tradesmen to mark the north-seeking end by a transversal file-mark.)

Introductory Diagram explaining the representations of Magnets.

Poles of Red Magnetism, seeking the North.



Poles of Blue Magnetism, seeking the South.

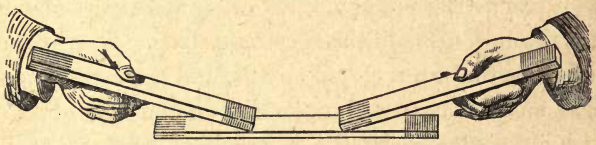
A horse-shoe magnet is merely a straight magnet bent into the horse-shoe form: it will be shewn hereafter that the properties of the two ends differ in the same manner as those of the ends of a straight magnet.

8. *Method of magnetizing steel bars, and of preserving their magnetic power.*

It is not easy to say how artificial magnets were formed in the first instance. Probably they may have been derived from the natural magnet; whose attractive properties, and whose power of producing temporary

magnetism in iron, have been known from a very distant age. But, magnets having been once formed, there is little difficulty in forming other magnets from them. The most convenient process for magnetizing a steel bar or compass-needle, &c. is that known by the name "double touch." It requires the use of two magnets. The bar which is to be magnetized being laid horizontally, with some slight band to prevent it from moving, the operator takes one magnet in his right hand with (say) the red end downwards, and one in his left hand with the blue end downwards (or both in the opposite positions, according to the nature of the magnetism which he wishes to impart), and, touching the bar with the ends of the magnets near the middle of its length

Fig. 1.



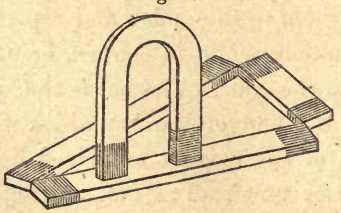
(see Figure 1), he draws the two magnets simultaneously to the two ends of the bar (constantly maintaining the contact) till they slip off. He raises the magnets, again places them in contact with the middle of the bar, and again slides them to the ends: and repeats this operation: the motion, while in contact, being always from the middle to the ends. The bar is thus converted into a magnet: the end of the bar which was touched by the red end of the magnet employed becomes a blue end; and *vice versâ*. The magnetizing magnets, in

general, suffer no deterioration from this employment. The new steel magnet will retain its magnetism through an indefinitely long period: its permanency depending greatly on the quality of the steel. The steel best adapted for large magnets is that which is best for fine cutlery: and the steel should be perfectly hard through the whole length of the bar. For compass-needles, the same steel at spring-temper is found more advantageous.

It is possible, by various contrivances, as for instance by holding both the hand-magnets with the red ends downwards or both with the blue ends downwards, to create a magnet with a concentration of magnetism in the middle of its length. Such magnets however are useless, and we shall not further notice them.

Some artists prefer the following method of magnetizing simultaneously three or more bars. The bars are laid so as to form a closed circuit, and a powerful horse-shoe magnet is placed upon any one bar with both its ends in contact with the bar, and is carefully carried thus round the whole circuit of bars, always in contact, and with the same end of the horse-shoe magnet always preceding. (See Figure 2.) The circuit is repeated

Fig. 2.



several times without lifting the horse-shoe magnet. The red and blue ends of the resulting magnets are respectively opposed in position to those of the horse-shoe magnet. (On the mode of distinguishing the ends of a horse-shoe magnet, we shall speak shortly.)

In either process, after a time, a limit to the intensity of the communicated magnetism is reached. This is usually expressed by the phrase "magnetized to saturation."

The steel which is the most valuable for retention of magnetism is also the most favourable for reception of a strong dose of magnetism.

For preserving the magnets with full magnetic intensity, it is found prudent to place them side by side with their red and blue ends in opposite positions, and to connect the opposite ends (the red end of each with the blue end of the other) by pieces of iron in contact with both.

Valuable information connected with this subject will be found in the *Encyclopædia Metropolitana*, article *Magnetism*.

9. *The terrestrial force upon a magnet is a Couple: the red end is drawn towards the north, the blue end towards the south, with equal forces. First Law of Magnetism, the Duality of Powers.*

If a magnet, on which the Earth's directive power is strong, be suspended by a very long suspension-thread, and the position of the thread be noted; if then the magnet be removed and a lump of lead of equal weight

be substituted for it; the suspension thread takes exactly the same position as before. This shews that, upon the whole, there is no horizontal force tending to produce a motion of translation of the magnet; and therefore, if there is one force tending to draw the red end towards the north, there is an equal force tending to draw the blue end towards the south.

If a small magnet, as a compass-needle, be supported by two small pieces of cork and floated on water, it speedily takes the north-and-south position, but it shews no disposition to approach any side of the basin.

These experiments are very important. They shew either that there are different attractions from different parts of the Earth upon different parts of the magnet, or that attraction of the Earth on one part of the magnet is accompanied with equal repulsion on another part. We shall find that, without negativing the first of these suppositions, other experiments shew that the second is universally true: that attraction on one part of a magnet is universally accompanied with repulsion on another part. And thus we arrive at the *first* important law of magnetism, the "Duality of Powers." It is this duality which essentially distinguishes the force of magnetism from that of gravitation: in other respects, it will be seen, there is much similarity of their laws.

10. *Action of one magnet upon another. Second Law of Magnetism. There is attraction between dissimilar ends, repulsion between similar ends. This is exhibited in various ways. Idea of poles. When one*

magnet disturbs a compass, another magnet may be so placed as to neutralize the disturbance. Poles of a horse-shoe magnet.

The experiments proving these general laws are the easiest of all. Turn the red end of a magnet held in the hand towards the red end of a suspended needle or compass-needle, and it repels the needle's red end. In like manner, the blue end of the hand-magnet repels the blue end of the needle. On the contrary, turn the red end of the hand-magnet towards the blue end of the needle, and it attracts the needle's blue end; and in like manner, the blue end of the hand-magnet attracts the red end of the needle.

The same principle may be exhibited in various forms. If the red end of the hand-magnet be pointed, from a distance, at right angles towards the middle of the needle, it attracts the blue end and repels the red end; shewing (in addition to the law which we have before us) that the ends of a magnet can act obliquely: an important remark on which we will speak further. If the hand-magnet be placed, at a distance, with its center in the line of the needle produced, and its direction transversal to that of the needle, it disturbs the needle according to the same law. If the hand-magnet be placed with its center vertically above or vertically below the center of the needle, and its direction transversal to that of the needle, the same remark holds. All these experiments lead to the Second Law of Magnetism; that there is repulsion

between magnetisms of similar character and attraction between magnetisms of dissimilar character.

An additional result, of some importance, is gained by holding the hand-magnet in a vertical direction and bringing it sideways towards one end of the needle. It will be found that the energy of the attraction (or repulsion, as the case may be) varies as the hand-magnet is moved up and down; and that it is greatest when a part of the hand-magnet near to its end but not at its end (distant from it perhaps by $\frac{1}{12}$ of the magnet's whole length) is nearest to the needle. This suggests the idea that the whole of the magnetism peculiar to that end of the magnet is collected into that one point: and that point is called a "Pole." But in fact it is found that, in varying the experiment, no point can be fixed on as strictly corresponding to this idea of a pole; still the language and the idea are so convenient that we shall make use of them, in general description, and even in some investigations.

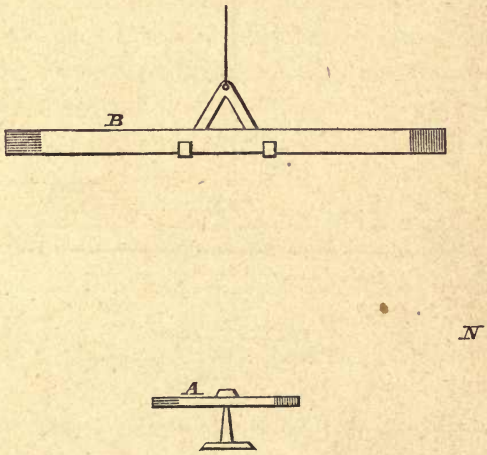
It is easily found that the effect of one magnet may be neutralized by that of another magnet. Thus, if one magnet be below the needle, a similar magnet above the needle with its poles in opposite positions will neutralize it. The reader will have no difficulty in varying this experiment, so as to make it applicable to the other cases of magnetic disturbance.

If a horse-shoe magnet be held in a vertical position, and if its ends be separately presented sideways to a suspended magnet, it will be found that they possess

respectively red and blue poles, exactly similar to those of a straight bar-magnet.

11. *The action of the Earth is exactly the same as that of a large magnet, whose red end is on the south side and whose blue end is on the north side. For experimental purposes, the Earth's action may be neutralized over a large space. Or its action on a special magnet may be rendered insensible by use of the astatic needle.*

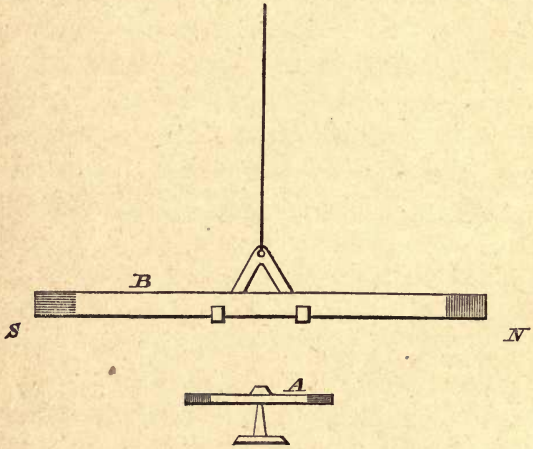
Fig. 3.



In Figure 3, suppose the needle *A* to be turning freely on a fine point and the magnet *B* to be delicately suspended above it, both magnets taking the position given to them by the Earth's magnetic power, and therefore parallel, with their red ends pointing to the north. In this state, the needle *A* is maintained vigor-

ously in its position; and, if it is drawn aside for a moment, it returns rapidly to that position. Lower *B* gradually: at a certain elevation of *B*, the needle *A* will become indifferent to position, and if drawn aside will not return to its former direction. Lower *B* still more, and *A* will reverse its position, its red end pointing to the south, as in Figure 4.

Fig. 4.



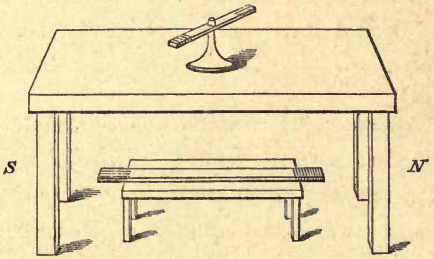
It is evident here that, at the second or intermediate position of *B*, the action of *B* is sensibly neutralized by the Earth's action. But, as we have remarked in the last article, the action of *B* may be neutralized by that of another magnet, at a proper distance, with its red pole to the south. Consequently, the Earth's action is exactly the same as that of a magnet whose red pole is south, and for magnetic

purposes the Earth may be represented by such a magnet.

The importance of this inference for theories of magnetism cannot be over-estimated. It shews, not only that the Earth's red and blue poles must be considered to be on the south and north sides, but also that the quality of the Earth's magnetism is the same as that of a steel magnet.

Advantage may be taken of this principle, in experiments, for removing terrestrial influence. If, as in Figure 5, a large magnet be placed at a proper distance below a table, magnetic experiments may be performed upon that table without disturbance by terrestrial magnetism.

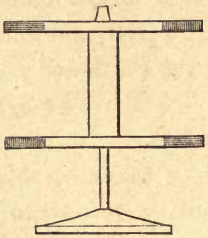
Fig. 5.



There is however another way of neutralizing the Earth's action, by use of the "astatic needle." This instrument, represented in Figure 6, consists of two needles of equal magnetic power, attached to the same central axis, with their poles in opposite positions. In this state, the action of the Earth on one needle is

balanced by its action on the other, and the united frame is indifferent to terrestrial magnetism. But one

Fig. 6.



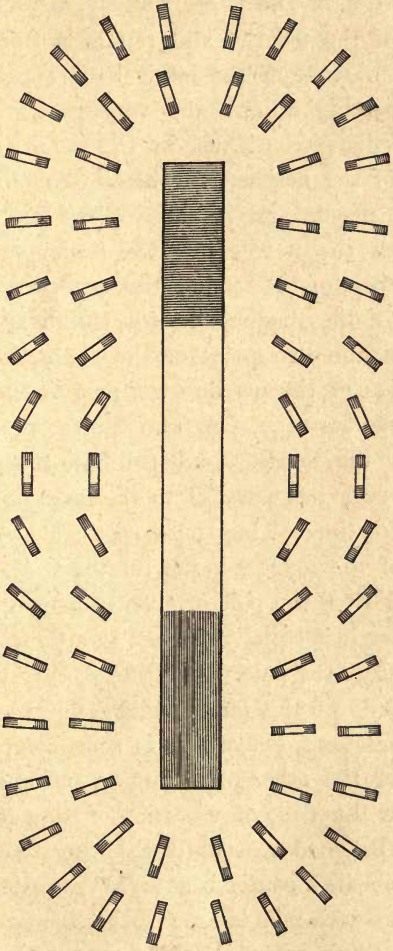
of the needles may be brought so near to the magnet whose force is to be tried that the comparative influence on the more distant needle may sometimes be neglected; and the experiments on the action of the magnet on the nearer needle will not differ much from what they would have been if terrestrial magnetism did not exist.

12. *Experimental examination of the action of a large magnet on a small needle. Third Law of Magnetism; the magnetism collected in or near each pole of a magnet acts (as to sense) equally in all directions.*

Underneath a table, let a large magnet be placed with its red pole north, at such a distance (determined by trial with a small needle on the table) that on the surface of the table the Earth's magnetism is sensibly neutralized. Place in that region a magnet of moderate size, carry round it a small compass, and register the positions of its needle. A series of directions is

obtained similar to those in Figure 7 (which is drawn

Fig. 7.



from actual experiment). It will be evident here

that the direction in which the red pole (for instance) of the needle is drawn is everywhere determined by the composition of two forces, namely, attraction to the blue pole of the magnet and repulsion from the red pole: the influence of the more distant pole (whichever it may be) diminishing very rapidly with the increase of distance. Thus, in the neighbourhood of each pole of the magnet, the attractive force on one pole of the needle and repulsive force on the other sensibly draw the needle into the same position as if the distant pole of the magnet did not exist; opposite the middle of the magnet's length, the distances of the needle from the two poles of the magnet are equal, the attraction of the needle's red pole to the magnet's blue pole and its repulsion from the red pole (and the opposite for the needle's blue pole) are sensibly equal, and the needle lies parallel to the magnet but in the opposite direction. Thus it is seen that the action of a pole of the magnet is not limited to the direction longitudinal from the pole or even transversal from the pole, but that it is equally distinct in a direction nearly backwards from the pole. It is not so easy to judge of the magnitude of the force which one pole exerts in different directions, because it is soon complicated by the effect of the other pole: but, on trying it at small distances by the time of vibration of the needle, there appears to be good reason for thinking that the force when the needle's center is at a transversal separation from the magnet's pole is exactly the same as the force when the needle's center is at a longitudinal separation

from the magnet's pole. The experiment of vibration may be extended to a position of the needle much nearer to the center of the magnet than is the magnet's pole. And thus we arrive at the Third Law of Magnetism, that the magnetism collected in or near the pole of a magnet acts equally, as to sense, in all directions. In this respect, magnetism resembles gravitation. (The law of force, as depending on the distance, will be a subject of future inquiry.)

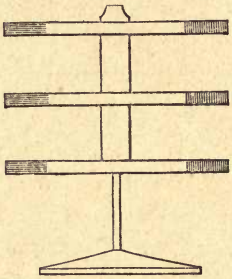
13. *Experimental proof of the Fourth Law of Magnetism, that the attraction or repulsion (as the case may be) between two masses of magnetism, estimated as a statical force, is proportional to the product of their magnetic energies. Definition of the units of the elements used in succeeding investigations; novel unit of statical pressure.*

Without at present giving an algebraical or numerical definition of magnetic energy, it may be understood as being, in needles of similar form, proportional to the force by which, under the action of terrestrial magnetism, the red end is drawn to the north and the blue end to the south. The successive steps of experiment bearing upon the law now under consideration will be the following:

(*a*) Provide a light axis capable of receiving, at pleasure, one, two, three, or more needles, made as similar as possible, and charged as nearly as possible with the same amount of magnetism. (This is easily

verified, by their power of deflecting a compass-needle.) The apparatus is represented in Figure 8. Suspend the

Fig. 8.



axis delicately; load it with each of the needles in succession, one at a time; observe the time of vibration as produced by terrestrial magnetism; and, if they differ slightly, take the mean. Then place all the needles on the axis, and it will be found that the time of vibration is the same as that mean. This shews that the terrestrial magnetic statical force on the assemblage of needles bears the same relation to the moment of inertia of the assemblage as that which existed for a single one: and therefore that the terrestrial magnetic statical force is proportional to the number of similar magnets on which it acts.

(*b*) The direction in which terrestrial magnetism acts being known, and a line being drawn through the center of the axis at right angles to that direction, place in that line a magnet (either similar to one of the needles, or of any other form and magnitude) which

will deflect the needles mounted on the axis. As every one of the actions between the respective poles is a statical action, and as the mean of the actions on the nearer pole and the further pole of the needle will be sensibly the same as if each was at the needle's center, the trigonometrical tangent of deflection will be the proportion of the statical force exerted by the magnet to the statical force exerted by the Earth. Now the fact of experiment is, that the deflection produced by the external magnet in the assemblage of needles is exactly the same as the deflection produced in a single needle. And therefore the proportion of the statical force exerted by the external magnet to the statical force exerted by the Earth is the same in both cases. But, as we have seen, the statical force exerted by the Earth is proportioned to the number of needles or to the sum of magnetic energies of the needles; and therefore the statical force exerted by the external magnet is proportional to the sum of magnetic energies of the needles. And the algebraical expression for that statical force must contain that sum of magnetic energies of the disturbed needles as factor. The same rule holds good with regard to gravitation.

It may at first appear strange that the pull exerted by a magnet upon several needles is greater than the pull exerted upon a single needle, and that in fact a new equal pull is ready to act upon every new equal needle exposed to it. But the fact is so; and it is analogous to the gravitation-attraction exercised by a planet upon several satellites, in which the force upon

one satellite is not diminished by the circumstance that the same planet is acting also upon another satellite.

(*c*) Use the apparatus of Figure 8 as a deflecting apparatus, to deflect from its ordinary position a compass-needle. Place the axis of Figure 8 in the direction magnetic E. or W. from the center of the compass: and mount successively upon it one needle, two needles, three needles. If the single needles continue in their combined state each to exercise the same action as when it is alone, so that the whole statical pull on the compass-needle is successively represented by 1, 2, 3, then the trigonometrical tangents of the angles of deflection of the compass-needle will be in the successive proportions of 1, 2, 3. And the fact, in experiment, is so. It follows from this that the statical force exerted by the assemblage of needles is proportional to the sum of the statical forces exerted by each single needle: that is, it is proportional to the sum of the magnetic energies of these needles. And therefore, the expression for the statical force exerted must contain the sum of the energies of the disturbing needles as factor.

(This might have been inferred from the conclusion of (*b*), or *vice versâ*, by assuming the equality of statical action and reaction. But in a matter of such fundamental importance, it appears well to establish each proportion by independent experiment.)

(*d*) Combining the results of (*b*) and (*c*), it will be seen that the algebraical expression for the statical

force exerted between the two magnetic systems must contain as factor the product of the energies of the two systems.

The experiments cited in this Article have been carefully verified by the writer of this Treatise.

It is necessary now to fix with precision the units of the different elements which we have to employ. For the unit of time, 1 second of mean solar time is universally adopted: for the unit of measure of length, 1 foot is commonly used in England, and 1 millimètre by the nations which adopt the Metrical system: for the measure of mass, reference is made to weight, and the received units are, 1 grain in England, and 1 milligramme in the Metrical system. For the measure of statical force, it is found convenient to depart from the custom usually followed in mechanical investigations (in which the unit of pressure is considered to be the pressure produced by a unit of mass under the action of terrestrial gravity), and to adopt, instead, that pressure which, acting through the time 1 upon the mass 1, would produce in it the velocity 1. (This unit, in English experiments, is about $\frac{1}{32 \cdot 1}$ of the ordinary unit of pressure.) This selection of unit of pressure amounts to the same as saying that the unit of accelerative force will be that which produces the velocity 1 in the time 1.

SECTION III.

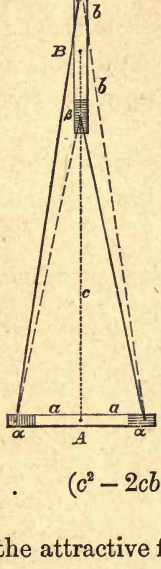
ALGEBRAICAL INVESTIGATIONS OF THE ACTION OF ONE MAGNET UPON ANOTHER; THE MAGNETS BEING IN THE SAME PLANE, AND THE FORCE OF ATTRACTION OR REPULSION VARYING AS A POWER OF THE DISTANCE.

14. *The disturbing magnet presents the center of one side at right angles to the disturbed magnet (a position which we shall hereafter term "broadside-on"), and the disturbed magnet presents one end to the center of the disturbing magnet (a position which we shall call "end-on"); the magnetical energies are supposed to be collected in the poles, and the attractive or repulsive force to vary inversely as the* m*th power of the distance: to find the angular momentum impressed on the disturbed magnet. Attractions will be represented in the diagrams by continuous lines, repulsions by interrupted lines.*

In Figure 9, suppose that we require the angular

momentum which A produces on B. The continuous lines denote attraction; the interrupted lines denote repulsion. Let $2a$ and $2b$ be the lengths of the two magnets as measured from pole to pole: α and β the magnetic energies at the poles (meaning by this that the attraction or repulsion will be expressed by $\alpha\beta \times \{\text{distance}\}^{-m}$); c the distance between the centers of the magnets, which is supposed to be considerably greater than a or b. Then the distance from the blue pole of A to the red pole of B is

Fig. 9.



$$(c^2 - 2cb + a^2 + b^2)^{\frac{1}{2}} = c\left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{1}{2}} :$$

the attractive force is

$$\alpha\beta \times c^{-m} \times \left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{-\frac{m}{2}} :$$

the resolved part of this, drawing the red pole of B to the right, is

$$\alpha\beta \times a.c^{-m-1} \times \left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} ;$$

a similar term is obtained from the repulsion of the red pole of A on the red pole of B: and the whole angular momentum which they impress on B is

$$2\alpha\beta \times ab \,.\, c^{-m-1} \times \left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}},$$

tending to turn it in the direction opposite to the motion of watch-hands. Similarly, the momentum obtained from the action of the two poles of A upon the blue pole of B is

$$2\alpha\beta \times ab \,.\, c^{-m-1} \times \left(1 + \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}},$$

in the same direction as the former. And the whole angular momentum, opposite to watch-hands, is

$$2\alpha\beta \times ab \,.\, c^{-m-1} \times \left\{ \left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} + \left(1 + \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} \right\}.$$

It is a great convenience, connected with the assumption that c is large in proportion to a or b, that we can at once proceed to expand these expressions in terms with progressive powers of c in the denominator, stopping at a definite power of c. It will be found sufficient, for future use, to stop with c^{-2} in the expansions of the brackets. Thus we shall have

$$\left(1 - \frac{2b}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} = 1 - \frac{m+1}{2}\left(-\frac{2b}{c} + \frac{a^2+b^2}{c^2}\right) + \frac{m+1 \,.\, m+3}{2.4} \cdot \frac{4b^2}{c^2};$$

$$\left(1+\frac{2b}{c}+\frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} = 1-\frac{m+1}{2}\left(+\frac{2b}{c}+\frac{a^2+b^2}{c^2}\right) + \frac{m+1\,.\,m+3}{2\,.\,4}\cdot\frac{4b^2}{c^2};$$

$$\text{sum} = 2-(m+1)\,.\,\frac{a^2+b^2}{c^2}+(m+1)\,(m+3)\,\frac{b^2}{c^2}.$$

The whole angular momentum

$$= 2\alpha\beta\times ab\times[2c^{-m-1}+(m+1).c^{-m-3}\{-a^2+(m+2)\,.\,b^2\}],$$

$$\text{or} = 4\alpha\beta\times ab\times[c^{-m-1}+\frac{m+1}{2}.c^{-m-3}\{-a^2+(m+2).b^2\}].$$

15. *In the same arrangement, to find the tendency of the action of* A *to produce a motion of translation of* B.

It is obvious that there is no tendency to carry B to or from A. But there is a tendency to carry it to the right. The forces on the two poles of B are the same as those just found, but that on the blue pole must be subtracted from that on the red pole. The efficient force therefore is

$$2\alpha\beta\times a\,.\,c^{-m-1}\times\left\{\left(1-\frac{2b}{c}+\frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} - \left(1+\frac{2b}{c}+\frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}}\right\}.$$

Expanding as before, this becomes

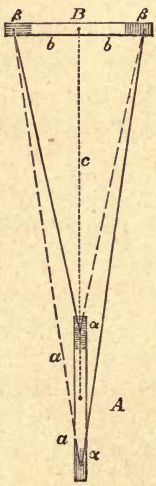
$$2\alpha\beta\times a\,.\,c^{-m-1}\times 2(m+1)\frac{b}{c}$$

$$= 4\alpha\beta\times ab\times(m+1)c^{-m-2}.$$

It is important to observe here that the negative power of c is greater than in the expression found in Article 14. The force which would produce the first or principal term in the expression for angular momentum is $4\alpha\beta \times a \times c^{-m-1}$. The proportion of the force of translation now found to that force is $\frac{b}{c}$. If then c be much larger than b, the force tending to produce translation is much smaller than the force producing angular momentum.

16. *The disturbing magnet is end-on towards the center of the disturbed magnet, which is broadside-on: to find the angular momentum impressed.*

Fig. 10.



The notation for Figure 10 being the same as for

Figure 9, it will be seen that the resolved part of the force which the red pole of A impresses on the red pole of B tending to turn it against watch-hands is

$$\alpha\beta \times (c-a) \times (c^2 - 2ca + a^2 + b^2)^{\frac{-m-1}{2}},$$

$$\text{or } \alpha\beta \times c^{-m} \times \left(1 - \frac{a}{c}\right) \times \left(1 - \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}};$$

which produces the angular momentum

$$\alpha\beta \times b \,.\, c^{-m} \times \left(1 - \frac{a}{c}\right) \times \left(1 - \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}}.$$

A similar term is produced by the attraction of the red pole of A on the blue pole of B: thus the whole angular momentum opposite watch-hands produced by the red pole of A is

$$2\alpha\beta \times b \,.\, c^{-m} \times \left(1 - \frac{a}{c}\right) \times \left(1 - \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}}.$$

The momentum produced by the blue pole of A is found in like manner to be

$$-2\alpha\beta \times b \,.\, c^{-m} \times \left(1 + \frac{a}{c}\right) \times \left(1 + \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}};$$

and therefore the entire momentum opposite watch-hands is

$$2\alpha\beta \times b \,.\, c^{-m} \times \left\{\left(1 - \frac{a}{c}\right) \times \left(1 - \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}} - \left(1 + \frac{a}{c}\right) \times \left(1 + \frac{2a}{c} + \frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}}\right\}.$$

In expanding these brackets, it will quickly be found that, in order to secure the same accuracy as in the development of Article 14, it is necessary to use the binomial theorem one step further:

therefore $$\left(1-\frac{2a}{c}+\frac{a^2+b^2}{c^2}\right)^{\frac{-m-1}{2}}$$

$$=1-\frac{m+1}{2}\left(-\frac{2a}{c}+\frac{a^2+b^2}{c^2}\right)+\frac{m+1.m+3}{2.4}\left(-\frac{2a}{c}+\frac{a^2+b^2}{c^2}\right)^2$$

$$-\frac{m+1.m+3.m+5}{2.4.6}\left(-\frac{2a}{c}+\frac{a^2+b^2}{c^2}\right)^3$$

$$=1+(m+1)\frac{a}{c}+\frac{m+1}{2}\left\{(m+2)\frac{a^2}{c^2}-\frac{b^2}{c^2}\right\}$$

$$+\frac{m+1.m+3}{6}\left\{(m+2)\frac{a^3}{c^3}-\frac{3ab^2}{c^3}\right\}.$$

Multiply this by $\left(1-\frac{a}{c}\right)$, and we obtain

$$1+m\frac{a}{c}+\frac{m.m+1}{2}\cdot\frac{a^2}{c^2}-\frac{m+1}{2}\cdot\frac{b^2}{c^2}+\frac{m.m+1.m+2}{2.3}\cdot\frac{a^3}{c^3}$$

$$-\frac{m.m+1}{2}\cdot\frac{ab^2}{c^3}.$$

Similarly the second bracket is

$$1-m\frac{a}{c}+\frac{m.m+1}{2}\cdot\frac{a^2}{c^2}-\frac{m+1}{2}\cdot\frac{b^2}{c^2}-\frac{m.m+1.m+2}{2.3}\cdot\frac{a^3}{c^3}$$

$$+\frac{m.m+1}{2}\cdot\frac{ab^2}{c^3};$$

and the whole large bracket is

$$2m\frac{a}{c}+\frac{m.m+1.m+2}{3}\cdot\frac{a^3}{c^3}-m.(m+1).\frac{ab^2}{c^3};$$

and the entire angular momentum opposite watch-hands is

$$2\alpha\beta \times b \,.\, c^{-m} \times \left\{ 2m \frac{a}{c} + m \,.\, (m+1) \left(\frac{m+2}{3} \cdot \frac{a^3}{c^3} - \frac{ab^2}{c^3} \right) \right\},$$

or

$$4\alpha\beta \times ab \times \left\{ m \,.\, c^{-m-1} + \frac{m.(m+1)}{2} \,.\, c^{-m-3} \left(\frac{m+2}{3} a^2 - b^2 \right) \right\}.$$

On comparing this result with that of Article 14, it will be seen that the first or principal term here has the factor m, while that in Article 14 has the factor 1.

17. *In the same arrangement, to find the tendency of the action of* A *to produce a motion of translation of* B.

Here also it is seen that there is no tendency to carry B to or from A, but there is a tendency to carry it to the right. Both actions of the red pole of A tend to carry B to the right, and both actions of the blue pole of A tend to carry B to the left. The result is the same as that of Article 15, of which the circumstances are exactly reversed in this problem.

18. *To find the tendency of the action of* A *upon* B *in the simplest cases of parallelism of the two magnets.*

The cases to be considered are those represented in Figures 11, 12, 13, 14. In all these, there is no tendency to produce angular motion, or to produce motion of translation to the right or left. But there is tendency to produce motion of B to or from A. The student will easily verify the following results.

In Fig. 11 there is force, tending to produce a motion of translation of B from A, represented by

$$4\,(m+1)\,\alpha\beta \times ab\,.\,c^{-m-2}.$$

Fig. 11.

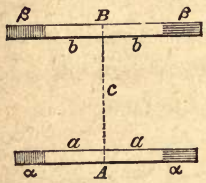


Fig. 12.

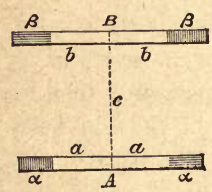


Fig. 13.

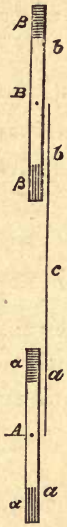
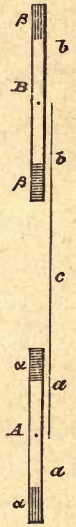


Fig. 14.



In Fig. 12, there is an equal force tending to produce motion of B towards A.

In Fig. 13, there is force tending to produce motion of B towards A, represented by

$$4m\,.\,(m+1)\,\alpha\beta \times ab\,.\,c^{-m-2}.$$

In Fig. 14, there is an equal force tending to produce motion of B from A.

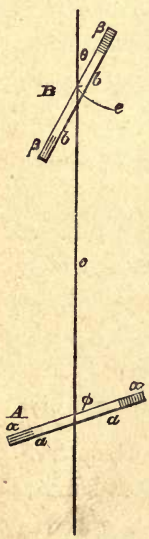
In all these cases, the index of c is $-m-2$, and the

result is subject to the same remark as those of Articles 15 and 17.

19. *Remarks on the computation of these quantities by the method of Potentials.*

The method of Potentials depends on the algebraical fact, representing a mechanical law like that of Virtual Velocity, that when a point $x'y'z'$ attracts a point xyz with a force R which is a function of the distance r between the points, the force in the direction x, or $R.\frac{x'-x}{r}$, can be expressed as $-R.\frac{dr}{dx}$: and therefore, if $R=\frac{dS}{dr}$, it can be expressed as $-\frac{dS}{dr}.\frac{dr}{dx}$ or $-\frac{dS}{dx}$: and so in the directions of the other co-ordinates. (Repulsion must be considered as negative attraction.) Here S is the Potential.

Fig. 15.



In the case of a needle B, let x be measured upwards on the paper and y to the right hand: if its semi-length is b inclined at an angle θ to the axis of x, and its center has for co-ordinates c in x and e in y, x will $=c+b\cos\theta$, $y=e+b\sin\theta$. These apply to the pole which is on the right hand of the diagram Fig. 15: for the opposite pole, b is negative. The form of the general theorem can then be conveniently altered, thus:—

(*a*) The force on the right-hand pole of B tending to increase x is

$$-\frac{dS}{dr}\cdot\frac{c+b\cos\theta-x'}{r}.$$

But since $r^2=(c+b\cos\theta-x')^2+(e+b\sin\theta-y')^2$, $\frac{dr}{dc}$ will $=\frac{c+b\cos\theta-x'}{r}$, and therefore the force tending to increase x will $=-\frac{dS}{dr}\cdot\frac{dr}{dc}$

$$=-\frac{dS}{dc}.$$

To estimate the whole force on the needle in the direction of x, the aggregate of the functions S for all the various attractions and repulsions on the two poles must be taken, and developed in the most convenient form (that proceeding by inverse powers of c will always be most convenient), and then it is only necessary to differentiate with respect to c.

(*b*) In like manner, the force tending to carry the needle to the right is $-\frac{dS}{de}$. But if we suppose the center of the needle B to be really on the axis of x, it is only necessary to retain e for the purpose of differentiation, and then, *after the differentiation*, e may be made $=0$. It is evident that, in this case, it is only necessary to develope S as far as the first power of e.

(*c*) The force at right angles to the length of B, tending to produce motion opposite to watch-hands, is

$$\text{force in } x\times\sin\theta-\text{force in } y\times\cos\theta,$$

or $-\dfrac{dS}{dr}\cdot\dfrac{1}{r}\{(c+b\cos\theta-x')\sin\theta-(e+b\sin\theta-y')\cos\theta\}$:

or $\dfrac{dS}{dr}\cdot\dfrac{1}{br}\{-(c+b\cos\theta-x')b\sin\theta+(e+b\sin\theta-y')b\cos\theta\}$,

or $\dfrac{dS}{dr}\cdot\dfrac{1}{br}\cdot r\dfrac{dr}{d\theta}$, or $\dfrac{1}{b}\dfrac{dS}{d\theta}$;

and the angular momentum opposite to watch-hands $=\dfrac{dS}{d\theta}$.

(*d*) Thus we have all the forces that we require, expressed in terms of a single function S; which we must now proceed to develope. If the force of attraction between a pole of magnet A and a pole of needle $B=\alpha\beta\times r^{-m}$, which must be taken for one part of $\dfrac{dS}{dr}$, then the corresponding part of $S=\dfrac{\alpha\beta}{m-1}\times r^{-m+1}$. Let the disturbing magnet A be inclined at the angle ϕ to the axis of x; then for the right-hand pole, $x'=a\cos\phi$, $y'=a\sin\phi$. And, supposing the similar poles of the two magnets to be on the same side, the complete value of S will be

$$\frac{\alpha\beta}{m-1}\times\left\{\begin{array}{l}+\{(c+b\cos\theta-a\cos\phi)^2+(e+b\sin\theta-a\sin\phi)^2\}^{\frac{-m+1}{2}}\\+\{(c-b\cos\theta+a\cos\phi)^2+(e-b\sin\theta+a\sin\phi)^2\}^{\frac{-m+1}{2}}\\-\{(c+b\cos\theta+a\cos\phi)^2+(e+b\sin\theta+a\sin\phi)^2\}^{\frac{-m+1}{2}}\\-\{(c-b\cos\theta-a\cos\phi)^2+(e-b\sin\theta-a\sin\phi)^2\}^{\frac{-m+1}{2}}\end{array}\right\}.$$

(*e*) For applying this method to specific cases, such as those of Articles 14, 15, 16, 17, 18, it is neces-

sary to begin by taking the ordinates c, e, θ in all their generality; and, *after differentiation*, to substitute the specific values of c, e, θ. The special investigations will be found to be easier.

20. *Investigation of the cases of Articles* 14, 15, 16, 17, 18, *on the supposition that magnetism is not confined to two poles of each magnet but is disseminated according to some law through its whole length, being entirely red magnetism on one side of the center and entirely blue on the other side, with similar laws of distribution. Definition of the "magnet-power" of a magnet.*

Let x be measured from the center of A towards the right-hand pole or upper pole (in the preceding diagrams), and let the amount of magnetic energy there, for the length δx, be $\alpha\delta x$. And let y be similarly measured for B, and let the amount of magnetic energy for the length δy be $\beta\delta y$. To the action of these two masses of magnetism, the investigations of Articles 14 to 19 will apply. But in the investigation for the action of a single pole of A on a single pole of B, certain terms appeared which disappeared in the aggregate. We will therefore adopt the results of the investigations as if those terms had never appeared at all.

(a) From Article 14, with the disturbing magnet broadside-on and the disturbed needle end-on, the angular momentum produced by these two masses is

$$\alpha\delta x.\beta\delta y \times xy \times \left[c^{-m-1} + \frac{m+1}{2}c^{-m-3}\{-x^2+(m+2).y^2\}\right];$$

or $c^{-m-1}.\alpha x\delta x.\beta y\delta y$

$$+c^{-m-3}\times\left(-\frac{m+1}{2}\alpha x^3\delta x.\beta y\delta y+\frac{m+1.m+2}{2}\alpha x\delta x.\beta y^3\delta y\right).$$

This is to be integrated through the whole length of A, from $x=-a$ to $x=+a$, and through the whole length of B, from $y=-b$ to $y=+b$. Now, remarking that x and y are absolutely independent, and that the limits of the integrations are absolutely independent, it will be seen at once that the double integral of each term is simply the product of the two single integrals related to the two variables x and y in that term, and thus the double integral becomes

$$c^{-m-1}.\int_x\alpha x.\int_y\beta y+c^{-m-3}\left\{-\frac{m+1}{2}\int_x\alpha x^3.\int_y\beta y\right.$$
$$\left.+\frac{m+1.m+2}{2}\int_x\alpha x.\int_y\beta y^3\right\}.$$

We shall call the definite integral $\int_x\alpha x$ "the magnet-power of A," and shall put the symbol A for it: and similarly, we shall call the definite integral $\int_y\beta y$ "the magnet-power of B," and shall use the symbol B for it. For the whole of the bracket, which is a function of definite integrals, we shall put K. (It is to be remarked that every one of these terms has a real value, inasmuch as, when the sign of x is changed, that is when we consider the other half of the magnet, the quality of the magnetism is changed, and therefore the sign of α is changed: and therefore the sign of αx or of

αx^3 is unaltered, and the integral throughout is a continual aggregation of quantities with the same sign: and so for βy and βy^3.) Then the expression for the angular momentum is

$$c^{-m-1}.AB + c^{-m-3}.K;$$

in which, when c is large, the second term is much smaller than the first. Succeeding terms would be multiplied by c^{-m-5} &c., and, by taking c large enough, may certainly be made insensible.

(*b*) Treating the result of Article 16 in the same way, and putting L for the bracket which then presents itself, we find that, with the disturbing magnet end-on and the disturbed needle broadside-on, the expression for the angular momentum is

$$c^{-m-1}.m\,AB + c^{-m-3}.L;$$

in which, when c is large, the second term is much smaller than the first.

(*c*) The reader's attention is particularly called to the circumstance that the principal term in the second case (disturbing magnet end-on) is m times as great as that in the first case (disturbing magnet broadside-on): and that in both the successive powers of c are $-m-1$, $-m-3$. Thus, if the law of attraction and repulsion of magnetic masses be the inverse square of the distance, or $m = 2$, then the principal term in the second case is double the principal term in the first case, and the successive indexes of c are $-3, -5$. It will be found that these remarks are of the utmost importance

in determining the law of magnetic attraction and repulsion.

21. *Investigation of the action of a magnet, whose poles are very widely separated, upon a needle whose center is in the line of these poles and whose axis is inclined to that line.*

Fig. 16.

In Figure 16, the repulsion of the magnetic red mass S upon the red mass $\beta\delta y$ at the point y of the needle is

$$S.\beta\delta y\,(c^2 + 2cy\cos\theta + y^2)^{\frac{-m}{2}};$$

which being resolved into a part acting endways of the needle and a part parallel to c gives for the latter part

$$S.\beta\delta y.c\,(c^2 + 2cy\cos\theta + y^2)^{\frac{-m-1}{2}},$$

which produces the angular momentum opposite watch-hands

$$y.\sin\theta.S.\beta\delta y.c\,(c^2+2cy.\cos\theta+y^2)^{\frac{-m-1}{2}}.$$

Now when c is very large, it will suffice to retain the first efficient term in the expansion, which gives

$$y.\sin\theta.S.\beta\delta y.c^{-m},$$

or

$$\sin\theta.S.\beta y\delta y.c^{-m}.$$

The integral of this, through the whole needle, will be

$$\sin\theta.SB.c^{-m}.$$

In like manner, the attraction of the mass of N of blue magnetism will produce an angular momentum in the same direction represented by

$$\sin\theta.NB.c'^{-m};$$

and the total angular momentum will be

$$\sin\theta.B.(S.c^{-m}+N.c'^{-m}).$$

Let the quantity in the bracket be called E: then the total angular momentum =

$$EB.\sin\theta.$$

It is evident that, under the action of this force, the needle B will vibrate according to the same law as a common pendulum.

It will be remarked that the conditions here supposed correspond to those of the Earth's horizontal magnetic influence as observable at any one place. We know nothing of the precise action of the Earth's magnetism, except that, for any one place, it may be

represented by the action of a blue mass on the north side and a red mass on the south side, or by one of these alone. The result at which we have arrived is analogous in form to that which we have obtained in other magnetical investigations; and, to preserve the analogy of language, we shall call E the Earth's magnet-power. It is important to remark that, on account of the assumed magnitude of c and c', the angular momentum here is expressed by a single term, a multiple of B, without any of the additional terms which occur in Article 20.

SECTION IV.

On Terrestrial Magnetism, as acting in the horizontal plane at each place of observation.

22. *Definition of Local Magnetic Meridian and Magnetic Variation or Declination; instruments and methods for ascertaining roughly the Declination: Azimuth Compass; observation of Sun's Azimuth, Variation Theodolite; Declination Charts.*

The 'Local Magnetic Meridian' is the direction, on the horizontal plane of any place of observation, which is taken by the compass-needle. The 'Magnetic Variation' (a very bad term, still commonly employed by nautical men, but for which, among men of science, the term 'Magnetic Declination' is usually substituted) is the angle made by the Local Magnetic Meridian with the Astronomical Meridian: it requires to have appended to it the word "east" or "west" as applied to the north point of the needle. Thus the Magnetic Declination at Greenwich at the present time is about 20° West: meaning that the north point of the magnetic needle points west of the astronomical north meridian by 20° nearly.

The general direction of the Local Magnetic Meridian may be obtained by merely observing the direction of the needle: but the Astronomical Meridian is not so obviously visible: and therefore the Declination cannot be easily found. Navigators however require the Declination in order to enable them to adapt their compass-steering to ordinary maps and charts. The

Fig. 17.

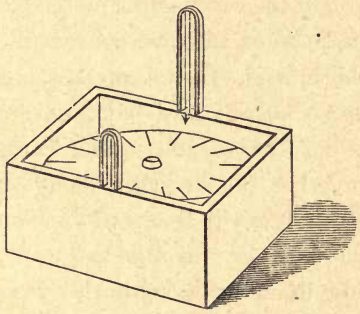
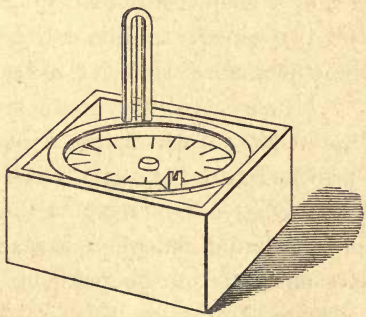


Fig. 17*.



element required was obtained by the use of an 'azimuth compass' represented in Figures 17 and 17*.

(The jimbal-rings, by which the compass-card retains its horizontality in all motions of the ship, are omitted in these Figures.) It must be remarked that in ships' compasses the compass-needle is never exhibited naked, but is inclosed in the thickness of a 'compass-card,' a circular card on which points of the compass and degrees of azimuth are engraved, and which, being firmly connected with the compass-needle, is so directed by the magnetic power of the earth acting on the compass-needle that the N, E, S, W on the card point truly to magnetic north, east, south, west. In the compass, nothing touches the circumference of this card, but there are, rising from the compass-box, two small frames carrying vertical wires; by directing the eye-view along the two wires and turning the box till that eye-view sees a distant object in the same line as the two wires, the line of wires is made to coincide with the direction of the object: and then the observer must read the graduations of the card which correspond to points in the box below the two wires. In this way he obtains the azimuth of the object as referred to the Local Magnetic Meridian.

In the best modern instruments, a horizontal ring is expressly provided to carry the vertical wire-frame: and, instead of having a wire next to the eye, a glass prism acting by internal reflection is placed there; so arranged, that one half of the pupil of the eye can observe the wire on the further side of the horizontal ring, and the distant object; and the other half of the pupil can see the graduations of the compass-card by

internal reflection in the prism. (Small instruments of this kind, called "prismatic compass," are to be obtained at little expence, and are very convenient.)

The azimuth of an object being obtained as referred to the Local Magnetic Meridian, the next point is to find its azimuth as referred to the Astronomical Meridian. The power of doing this depends on the choice of the object. Navigators almost invariably choose the rising or setting Sun. The latitude of the ship being known with sufficient accuracy for this purpose, and the Sun's declination and consequent distance from the celestial pole being taken from the Nautical Almanac, the solution of a quadrantal equation gives the Sun's azimuth at rising or setting as referred to the Astronomical Meridian. The difference between this and the azimuth as referred to the Local Magnetic Meridian gives the Magnetic Declination. (The poetic reader will find much of this operation correctly described in Falconer's "Shipwreck.")

For determinations on shore, a "Variation-theodolite" was sometimes used, consisting of a common theodolite adapted to the measure of horizontal angles, the axis of its telescope being so much raised that the telescope could be pointed to the pole-star, by which the reading of the horizontal circle for astronomical meridian could therefore be found; and that the telescope could also be pointed down to a compass-needle immediately below the theodolite-frame, to the ends of which the telescope-object-glass could be directed, and which ends were made distinctly visible by affixing

another lens on the telescope-object-glass; by which process the horizontal reading for magnetic meridian was found.

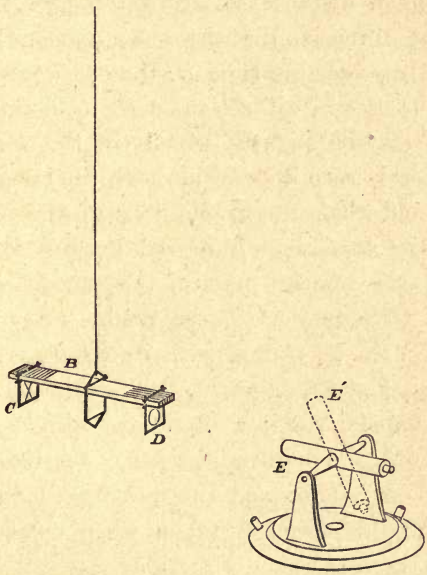
By such methods as these, "Variation Charts" or more properly "Declination Charts" have been prepared for the use of mariners, exhibiting the Compass-Declination on every part of our globe. And it is mainly by the use of these, in connexion with the compass, that ships are steered.

23. *More accurate method of determining the Local Magnetic Meridian; Reversed Telescope carried by Magnet; elimination of corrections for position of Magnetic Axis, and for torsion-power of suspension-thread.*

The most accurate apparatus for determining the Declination is however that represented in Figure 18, which is nearly copied from the instrument used at the Royal Observatory, Greenwich. It is founded essentially on the use of the "Reversed Telescope" or (as it is frequently but inappropriately called) the "Collimator." It is an ordinary optical theorem that, if parallel rays fall upon an object-glass, they will be made to converge to a point: and conversely, if rays diverge from a point at the focus of an object-glass and fall on that object-glass, they will emerge parallel. This quality of parallelism of rays possesses two important advantages. First, that the small object at the focus (which object will usually be a cross of wires) can be distinctly seen by use of a telescope, adapted to receive parallel rays, or rays coming from a distant

object, as a star. Second, that if that telescope be shifted laterally so as to receive the rays from one part of the parallel pencil instead of another, it still receives them in the same direction: so that minute adjustment of the lateral position of that telescope is not necessary.

Fig. 18.



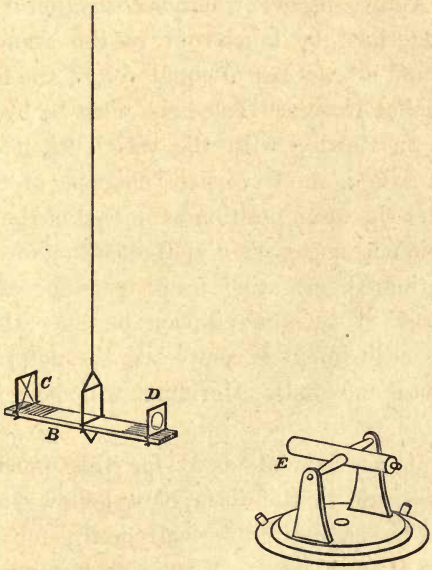
In Figure 18 therefore *B* represents a magnet suspended by a suspension-thread: *C* a small frame firmly attached to it, carrying a fine wire, or two wires crossing each other, and thus producing a fine point for observation: *D* another frame clamped to the magnet and carrying an object-glass whose focus is exactly at the place of the wires carried by *C*. The rays of light

diverging from the wire or mark in *C* and falling on the object-glass in *D* will emerge from it in a parallel pencil, every ray being parallel to that which passes from the mark *C* through the center of the object-glass in *D*: or parallel to the optical axis of the Reversed Telescope *CD*. A theodolite *E* must be placed at any convenient distance so that the object-glass of its telescope shall receive the whole or a part of the pencil of parallel rays coming from *D*: the eye, applied to the theodolite-telescope, will see the mark *C* distinctly: the theodolite may be slightly turned till the wire in its field of view is seen to coincide with the image of the mark *C*; and then the graduations of the horizontal circle of the theodolite indicated by the pointer or vernier of the rotatory part of the theodolite (which carries its telescope) are to be read. After this, the theodolite is to be turned with its telescope elevated at the proper angle, as at *E'*, to view the pole-star or other circumpolar stars; a meridional direction being thus obtained, the horizontal circle of the theodolite is to be read as before: and the difference between this reading and that formerly taken when observing the mark *C* is the magnetic declination.

But there are some delicate points requiring attention, which it may be well to describe here. First: the line along which our theodolite-telescope is directed is the optical axis of the Reversed Telescope *CD*. But the line which takes the direction of Terrestrial Magnetism is an undiscoverable line in the interior of the magnet, called its "magnetic axis." How can we be

certain that this magnetic axis is parallel to the optical axis of *CD*? The only thing that we can do is, so to arrange observations in a different state of the magnet that whatever error is produced by want of parallelism in the first case shall be exactly reversed in the second case; and the mean of results will then be free from error. It is merely necessary to invert the magnet in respect of the side on which it carries *CD*, as shewn in Figure 19; if the optical axis of *CD* pointed too much

Fig. 19.



to the west of magnetic north in the first case, it now points too much to the east of magnetic north in the second case, or *vice versâ*.

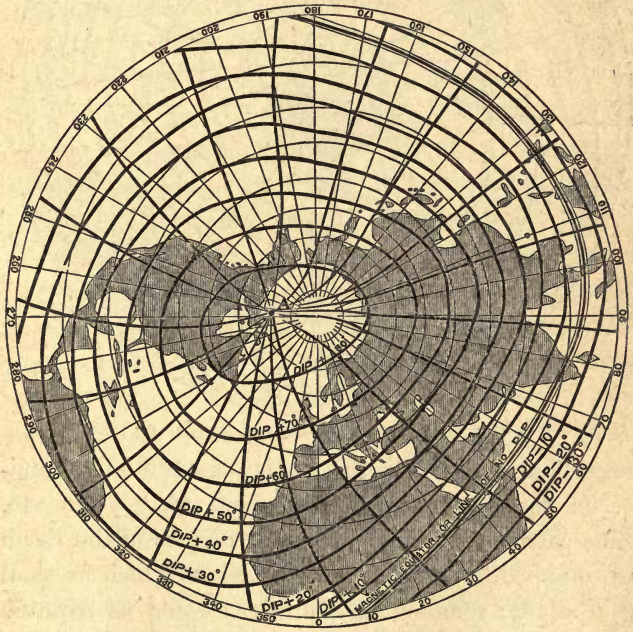
In practice there is another cause of error, from the torsion-power of the suspension-thread. If, in the observations above described, the thread has any tendency to turn the magnet horizontally, the results are certainly erroneous. To give the means of applying a mechanical correction, the suspension-thread must not be tied at its top to a fixed bar, but must be fastened to or hung on a hook or ring attached to a small frame which can be turned round horizontally. (Sometimes this means of rotation is given at the lower end of the thread.) And to discover whether correction is wanted, the magnet must be taken out of the frame which carries it, and a brass bar of equal weight and furnished with a similar Reversed Telescope must be hung in its place. If, on viewing with the theodolite, it is found that the mark in the Reversed Telescope of the brass bar occupies the same position as in that of the magnet, no correction is necessary. If it does not occupy the same position, the rotating frame carrying the hook must be turned till the position becomes the same. When this adjustment is completed, the determination of the Local Magnetic Meridian will be extremely accurate.

In some instruments used for this purpose, the magnet is made in the form of a hollow steel tube (which can be magnetized perfectly well), and the mark C and lens D are fixed in its interior, forming a veritable telescope.

24. *Terrestrial Magnetic Meridians; Historical physical changes in the system of Magnetic Meridians.*

The directions of the Local Magnetic Meridian being found at numerous points of the Earth, it is not difficult to trace through them a curve starting from any assumed point, and so drawn that in every part of its course its direction represents the direction of the Local Magnetic Meridian at that point. Such a curve may properly be called a Terrestrial Magnetic Meridian: and a number of these, at convenient intervals of

Fig. 20.

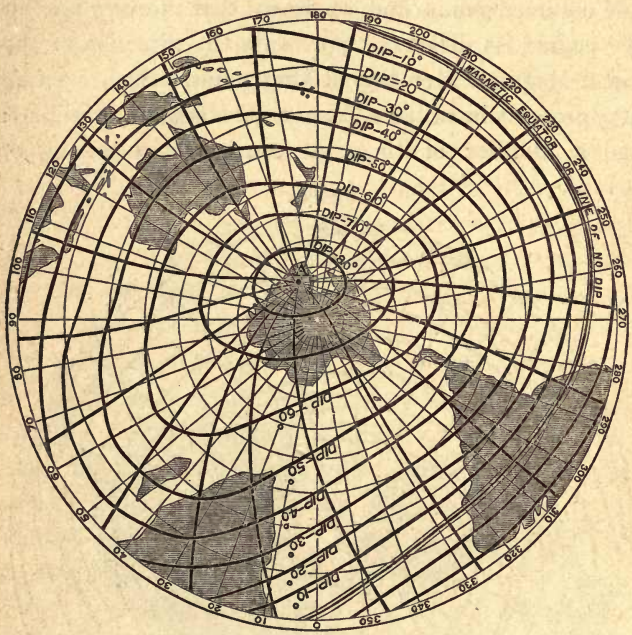


(A is the North Magnetic Pole. See Article 42.)

geographical longitude, may be advantageously used as a system of Terrestrial Magnetic Meridians. Such

a system is represented by the diverging strong lines in Figures 20 and 21, which are maps on the stereographic

Fig. 21.



(A is the South Magnetic Pole. See Article 42.)

projection. (Although this system is essentially founded on observations, collected about forty years ago, some portions of it in inaccessible regions of the Earth are supplied from a theory of Gauss's, to which we shall soon allude: some small alterations would be required to make it quite correct for the present date.) The forms of these Magnetic Meridians are very remarkable. None of them appears to be exactly a great circle.

They converge to a north pole, north of Hudson's Bay, and a south pole, in South Victoria: but these poles are not opposite. From E. longitude 70° to 150°, the northern parts agree nearly with Geographical Meridians: the same remark also applies to a large portion of the southern part in longitude 150°.

The system of Magnetic Meridians has undergone considerable changes in the times of modern accurate science. The southern point of Africa received from the Portuguese voyagers in the fifteenth century the name of L'Agulhas (the needle), because the direction of the compass-needle, or the Local Magnetic Meridian, coincided there with the Geographical Meridian: it now makes with it an angle of about 30°. In the sixteenth century, the compass-needle in Britain pointed east of north: it now points from 20° to 30° (in different parts of the British isles) west of north. At the present time, a change of the opposite character is going on: in 1819 the westerly declination at Greenwich was about 24° 23′, which was probably its maximum; in the last thirty years it has diminished from 23½° to 20°, nearly. It is believed that the magnetic poles are rotating round the geographical poles from East to West.

25. *Imperfect method of measuring the horizontal directive force of terrestrial magnetism at any locality, by vibrations of a magnetic needle. Correction for the torsion-power of the suspending-thread.*

In Article 21, it was found that the angular momentum, which the Earth's horizontal magnetic action im-

presses on a suspended needle whose axis makes the angle θ with the Local Magnetic Meridian, is $EB \sin \theta$: and if M be the moment of inertia of the needle, the equation which determines its angular motion will be $\frac{d^2\theta}{dt^2} = -\frac{EB}{M} \sin \theta$. If θ be very small, or if proper corrections be applied to the observed time of vibration (as for an ordinary pendulum) so as to reduce the time of vibration to what it would have been if the arc of vibration had been indefinitely small, we may use the equation $\frac{d^2\theta}{dt^2} = -\frac{EB}{M}\theta$: of which the solution is

$$\theta = C \sin\left(t\sqrt{\frac{EB}{M}} + D\right),$$

and the time T of a complete double vibration is $= 2\pi\sqrt{\frac{M}{EB}}$. This is a single equation containing two unknown quantities E and B, and neither of them can be determined from it.

If we repeat the experiment at a different time at the same station, or at any time at another station, where it may be presumed that the Earth's magnet-power is different, we shall have $T' = 2\pi\sqrt{\frac{M}{E'B}}$, which still gives no information. And if there is suspicion that the magnet-power B may have changed, there is no hope of arriving at any conclusion.

There is, however, one way of using the vibrating needle, from which an imperfect result may be obtained. If the vibrations be taken at a standard place (as

Greenwich); and if the needle be carried to other stations in rapid course, so that there is little reason to fear any change in its magnet-power; and if, for confirmation, the needle be again brought to the standard place: then we may obtain a certain result thus. Divide the square of the first equation above by the square of the second, and we have

$$\frac{T^2}{T'^2} = \frac{E'}{E}.$$

Thus we obtain the proportion of the Earth's magnet-power at one station to its magnet-power at another station, at the same time. But we get no positive information on the measure of the Earth's magnet-power at either station. And as we cannot suppose that the magnet-power of the needle will be unaltered through an unlimited time, we cannot use the experiment to determine whether the Earth's magnet-power at any station has altered with the lapse of time.

The principal value of this method is for very restricted local experiments.

In all applications of the method, we ought in strictness to take account of the torsion-power of the suspension-thread: as, on changing the suspension-thread, or in comparing observations where the difference of external magnetic action is great, the omission of that consideration may introduce important error. The torsion-power will be measured thus. Suppose the suspension-piece to be furnished with apparatus which admits of being turned round horizontally (as described

in Article 23). Note the position taken by the needle. Then turn the suspension-apparatus through a measurable angle, say 100°, and again note the position taken by the needle. If this position differs from the former, say by 1°, it shews that the torsion-power produced by a torsion of 100° is able to neutralize the Earth's action by the quantity corresponding to an inclination 1°; and therefore that the terrestrial action on the needle at any inclination θ is augmented by the power necessary to overcome $\frac{1}{100}$ part of terrestrial action at that angle θ; and that in fact the power which we measure is $\frac{101}{100}$ of the terrestrial power.

In this imperfect state the determination of the Earth's magnet-power remained till about the year 1835. And it does appear, at first view, impossible to obtain a numerical value, entirely freed from all dependence on weight of needle, quality of its steel, intensity of its magnetism, &c., for such an unprehensible element as terrestrial magnetism. A method however was introduced (first suggested, the writer believes, in an impracticable form by Poisson, and subsequently changed to an easy and accurate shape by Gauss) which solves the problem perfectly, and is now in daily use: it may well be considered as one of the most beautiful processes in experimental philosophy. To this process, after some preliminary experiments, and a determination of the last Law of Magnetism, we shall turn our attention.

26. *Experimental determination of the proportion of the magnet-power of a magnet to the Earth's horizontal*

magnet-power, by the method of deflexions. First process, the disturbing magnet applied broadside-on.

Fig. 22.

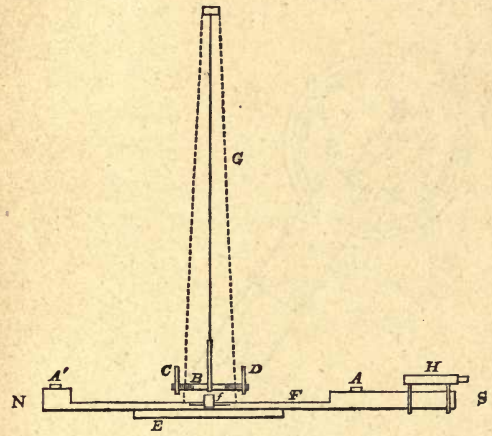
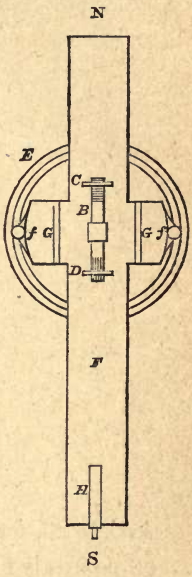


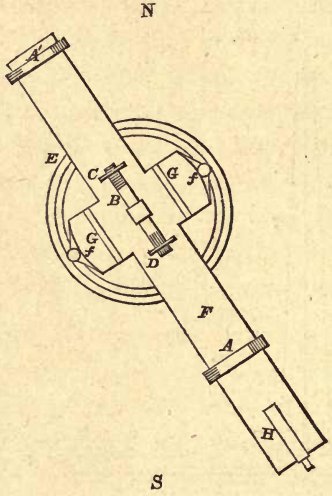
Fig. 23.



In Figure 22 is represented a side view of the deflexion-apparatus, and in Figures 23, 24, 25, horizontal plans of it under different circumstances. *E* is a graduated circle firmly fixed, preserving absolutely the same position in Figures 23, 24, 25. *F* is a bar turning horizontally round the centre of *E*, and having pointers or verniers *f*, *f*, for reading the graduations of *E*. *F* supports all the rest of the apparatus. From *F* there rise two uprights *G*, *G*, with a cross-bar at the top, to which is attached a suspension-thread

carrying the needle *B*. To *B* are attached a frame *C* with cross-wires and a frame *D* with object-glass, as in

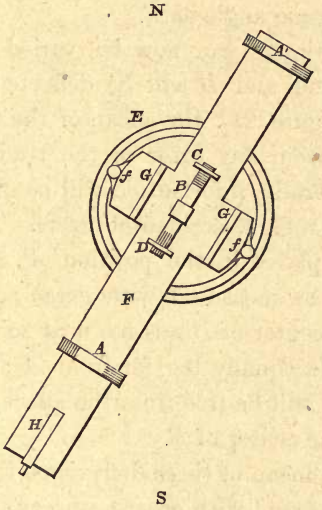
Fig. 24.



Article 23, forming a Reversed Telescope. *H* is a telescope firmly fixed to *F*, at the same height as *CD*, which can therefore see distinctly the mark *C*. *A* is the magnet for which the proportion of magnet-power to the Earth's horizontal magnet-power is to be found: it may sometimes be placed in the position *A'*: it is so supported that the view of the telescope *H* is not interrupted. In the Figure, the telescope is supposed to point to the north, but it may equally well point to the south. In the case now under consideration *A* is presented broadside-on to *B*. The observation begins without *A*, as in Figure 23; *F* is turned to such a position that,

while B takes its direction under the action of terrestrial magnetism only, the mark C is viewed with the telescope H and its image is made to coincide with the wire in H; in this state, the graduations of E under the

Fig. 25.



pointers f, f, are read. A is then mounted in its place, with its red end to the east, suppose, as in Figure 24: the north end of B immediately turns to the west: the bar F is turned carefully in the same direction, till the mark C is again seen by the telescope H, and its image coincides with the wire in H: then the graduations of E under f, f, are read. The difference between this reading of the graduations of E and the former reading is the angle through which B has been deflect-

ed by the action of A in the circumstances considered in the case (a) of Article 20, A being presented broad-side-on to B which is end-on. It is to be remarked that no new torsion of the suspension-thread is introduced by the rotation of B, because the suspension-bar, which is carried by the uprights on F, is made to rotate through the same angle as B.

The experiment can now be varied by reversing A, end for end, and B will be deflected the opposite way, as in Figure 25: the mean of the two deflexions will be free from any error in the reading of Figure 23: and, generally, information will be obtained which may assist to remove accidental errors. Also the magnet may be placed in the position A', whose distance from A can be measured with great accuracy: if in one case the center of B was too near to A, in the other case it will be equally too far from A'; and the mean of the results will be free from the effect of error in the position of the center of B.

From the mean of these deflexions, the true deflexion θ will be found with great accuracy. And now we remark that the needle B is kept in a definite angular position by the opposition of two forces whose angular moments must be equal. One is, the angular moment produced by A in the case (a) of Article 20: its value is

$$c^{-m-1} . AB + c^{-m-3} . K.$$

The other is, the angular moment produced by the Earth, as in Article 21: its value is

$$EB . \sin \theta.$$

The comparison of these (θ having such a sign that the forces are opposed) gives the equation

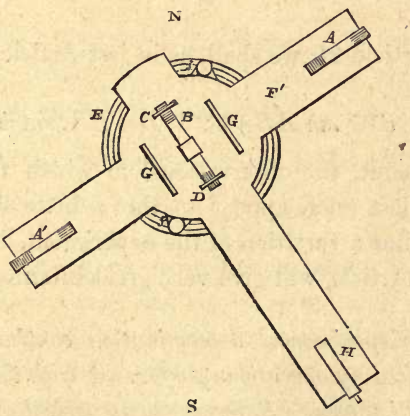
$$c^{-m-1} . AB + c^{-m-3} . K = EB . \sin\theta ;$$

in which it is to be remembered that the second term on the left side is small.

27. *Experimental determination continued. Second process, the disturbing magnet applied end-on.*

Here the form of the apparatus must be slightly varied. The reversed Telescope *CD*, and the telescope *H*, must occupy the same position as before: but the magnet *A* must be carried upon an arm *F* at right angles to *F*; see Figures 26, 27. For the same reasons as in

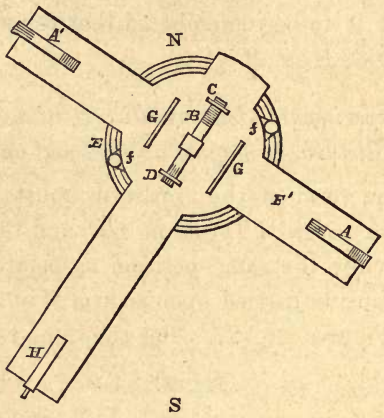
Fig. 26.



last Article, the magnet *A* will be used in the two positions shewn in these Figures, and also in two corresponding positions on the opposite half of the transversal

bar. Let ϕ be the mean of the angles thus obtained. Then, referring to the formulæ in case (*b*) of Article 20

Fig. 27.



and in Article 21, we shall, as in last Article, obtain the equation

$$c^{-m-1} \cdot m\,AB + c^{-m-3} \cdot L = EB \cdot \sin\phi.$$

To this point, the observations, to which this Article and the last refer, appear to throw little light on the subject: but a variation of the experiment, described in the next Article, will give very great information.

28. *Experimental determination continued. Each of the preceding operations performed with the needles at different distances. Reference to numerical observations. The discussion of the first process alone, the discussion of the second process alone, and the comparison of the two processes, shew independently that the attraction or repul-*

sion of magnetic masses is inversely as the square of the distance between them; Final Law of Magnetism.

Let the experiment of Article 26 be performed with two values of c (the distance between the centers of the magnets) which we shall call c_1 and c_2. Let the corresponding deflexions be θ_1 and θ_2. Then we have the two equations,

$$c_1^{-m-1} . AB + c_1^{-m-3} . K = EB . \sin \theta_1,$$

$$c_2^{-m-1} . AB + c_2^{-m-3} . K = EB . \sin \theta_2 ;$$

and nearly similar equations from Article 27, in each of which the second term on the left hand is to be small. And the question now to be considered is, What numerical value for m will enable us to satisfy that condition?

We shall approximately represent the state of the case, by neglecting the small terms, and thus we get the following approximate equations:

$$c_1^{-m-1} . AB = EB . \sin \theta_1 \; : \; c_2^{-m-1} . AB = EB . \sin \theta_2,$$

$$c_1^{-m-1} . m\,AB = EB . \sin \phi_1 \; : \; c_2^{-m-1} . m\,AB = EB . \sin \phi_2.$$

We must now appeal to actual experiment. A few observations bearing on these points were made by Gauss, but in the Greenwich Observations there are to be found about one hundred observations in which the disturbing needle was applied both broadside-on and end-on, and a far greater number in which it was applied end-on alone. We will take the observations of 1860, January 16.

Disturbing needle broadside-on.

$c_1 = 1{\cdot}0$ foot.	$c_2 = 1{\cdot}5$ foot.
$\theta_1 = 4^\circ . 29' . 20''.$	$\theta_2 = 1^\circ . 18' . 1''.$

Disturbing needle end-on.

$c_1 = 1{\cdot}0$ foot.	$c_2 = 1{\cdot}5$ foot.
$\phi_1 = 8^\circ . 33' . 1''.$	$\phi_2 = 2^\circ . 29' . 52''.$

It is to be remarked that

$$\left(\frac{c_2}{c_1}\right)^2 = (1{\cdot}5)^2 = 2{\cdot}25 \; : \; \left(\frac{c_2}{c_1}\right)^3 = 3{\cdot}375 \; : \; \left(\frac{c_2}{c_1}\right)^4 = 5{\cdot}0625.$$

Now first, dividing the first equation above by the second, we get the approximate equation

$$\frac{\sin\theta_1}{\sin\theta_2} = \left(\frac{c_2}{c_1}\right)^{m+1}.$$

Substituting the numerical values of $\sin\theta_1$ and $\sin\theta_2$, this becomes $3{\cdot}4490 = (1{\cdot}5)^{m+1}$. Looking at the powers of 1·5 placed above, it is impossible that $m+1$ can be 2 or 4, but the equation is so nearly true when $m+1=3$, (that is, 3·4490 approaches so near to 3·375) as to afford strong presumption that $m+1=3$.

Secondly, we obtain in the same way $\frac{\sin\phi_1}{\sin\phi_2}$ approximately $= \left(\frac{c_2}{c_1}\right)^{m+1}$. Substituting the numerical values of ϕ_1 and ϕ_2, this becomes $3{\cdot}4115 = (1{\cdot}5)^{m+1}$. This is very nearly correct if $m+1=3$.

Thirdly, if we divide the equation for ϕ_1 by that for θ_1 we obtain $\frac{\sin\phi_1}{\sin\theta_1} = m$. Substituting the numerical

values of the sines, this becomes $1{\cdot}8996 = m$. This affords a strong presumption that $m = 2$, agreeing with the deductions above.

Fourthly, if in like manner we divide the equation for ϕ_2 by that for θ_2, the numerical equation becomes $1{\cdot}9205 = m$.

It is impossible that any integer but 2 can represent the value of m: and, remarking that, in the omission of small terms, the equations above are not rigorously correct, the value $m = 2$ is at once assumed for further investigations; and this gives the Final Law of Magnetism, that attractions and repulsions of magnetic masses are inversely as the square of their distance.

29. *Inference as to the numerical value of the proportion of* A *to* E. *Remark on the unit of measure for* A *and* E.

Considering the law of inverse square of distance to be established, unless following phœnomena should contradict or modify it (none of which do so), it has been usual for experimenters to restrict themselves, for finding the ratio of AB to EB, to observations with disturbing needle end-on: partly because the apparatus is more convenient, partly because the deflexion with a given separation of needles is greater. The observations of deflexion being then taken with the needles at the two distances c_1 and c_2, we have the two following equations, which possess all the accuracy that is required for comparison with observation:

$$c_1^{-3} . 2AB + c_1^{-5} . L = EB . \sin \phi_1 ;$$
$$c_2^{-3} . 2AB + c_2^{-5} . L = EB . \sin \phi_2 .$$

Substituting the numerical values of c_1, c_2, ϕ_1, and ϕ_2, we have here two simple equations for determination of the two unknown quantities AB and L. Both will be expressed as multiples of EB. The quantity L is of no further use to us. But AB is now expressed by such a formula as $AB = k \times EB$: and therefore $A = k \times E$: k being a number given by the solution of the equations. Thus we have the magnet-power of the Earth expressed accurately as a multiple of the magnet-power of A.

Having thus acquired the power of separating the first term from the rest, in the expression for the angular momentum produced by the action of one magnet upon another, we may now conceive that first term only to be retained, in an action of which the idea will be serviceable to us. If c is 1 (that is, the unit of measure, whatever it may be), then the angular momentum, caused in B by the action of A broadside-on, is AB. Suppose now that we have two magnets S and S', exactly similar and equal, such that, using the letters for the magnet-powers as defined in Article 20, paragraph (a), $S = 1$, $S' = 1$: and suppose the distance of their centers to be $= 1$; then the angular momentum produced in S' by S, broadside-on, as depending on the first term of the formula, $= 1$. Such a magnet may well be conceived as a standard magnet. And when we obtain a numerical value A for the magnet-power of the needle A, it means that its action on S' at distance 1 is A times as great

as that of S at distance 1: and when we find that the Earth's magnet-power is $\frac{1}{k}\times$ that of A, it means that the Earth's action on S' is $\frac{A}{k}\times$ the action of S upon S' at distance 1.

30. *Investigation of the most advantageous proportion of the two distances of the disturbing magnet from the disturbed needle.*

If c_2 were very nearly $=c_1$, the two equations of last Article would be so nearly alike that the divisors giving the results of the equations would be small, and the possible errors in the measures of the angles θ or ϕ would produce enormous errors in the values found for the two unknown quantities. If c_2 were very large, the corresponding deviation would be very small, and the possible errors would bear a large proportion to the deviation. There is therefore a more advantageous proportion of c_2 to c_1, which it is our object now to ascertain.

The equations being of this form,

$$c_1^{-3}.x+c_1^{-5}.y=D_1,$$
$$c_2^{-3}.x+c_2^{-5}.y=D_2,$$

where the *actual* error of D_1 may be E_1, and the *actual* error of D_2 may be E_2; we find, by elimination of y,

$$(c_1^{-3}.c_2^{-5}-c_1^{-5}.c_2^{-3})\,x=c_2^{-5}.D_1-c_1^{-5}.D_2;$$

and

$$x=(c_1^{-3}.c_2^{-5}-c_1^{-5}.c_2^{-3})^{-1}\times c_2^{-5}.D_1-(c_1^{-3}.c_2^{-5}-c_1^{-5}.c_2^{-3})^{-1}\times c_1^{-5}.D_2,$$

subject to the *actual* error,

$$(c_1^{-3}.c_2^{-5}-c_1^{-5}.c_2^{-3})^{-1}\times c_2^{-5}.E_1-(c_1^{-3}.c_2^{-5}-c_1^{-5}.c_2^{-3})\times c_1^{-5}.E_2.$$

Let the *probable* error of D_1 be e_1, and the *probable* error of D_2 be e_2 (see the Author's Treatise on 'Errors of Observations,' Article 28): then the square of the *probable* error of the expression for x will be (see Articles 44 and 50 of the same treatise)

$$(c_1^{-3}.c_2^{-5} - c_1^{-5}.c_2^{-3})^{-2} \times c_2^{-10} \times e_1^2 + (c_1^{-3}.c_2^{-5} - c_1^{-5}.c_2^{-3})^{-2} \times c_1^{-10} \times e_2^2;$$

and if $e_1 = e_2 = e$, the square of the *probable* error of x

$$= (c_1^{-3}.c_2^{-5} - c_1^{-5}.c_2^{-3})^{-2} \times (c_1^{-10} + c_2^{-10}) \times e^2.$$

A value of c_2 is to be found which will make this minimum. Differentiating with respect to c_2 and making the differential coefficient = 0, and putting z for $\frac{c_2}{c_1}$, this equation is obtained,

$$5z^{10} - 3z^{12} + 2z^2 = 0;$$

of which the solution is

$$z \text{ or } \frac{c_2}{c_1} = 1{\cdot}32 \text{ very nearly.}$$

It is usual in the best modern instruments to make

$$c_2 = c_1 \times 1{\cdot}3.$$

31. *Incidental inference as to the effect of temperature upon the magnet-power of the disturbing magnet, and necessity for correction for temperature in various experiments.*

In considering the deflexion-experiments made under different atmospheric temperatures, it is found that the proportion of magnet-power of the disturbing magnet to that of the earth is smallest at the highest temperatures. To measure the amount of change for

1° of temperature, without depending on the assumption of uniformity of the Earth's magnet-power, the disturbing needle is inclosed in a copper box; and water, at different temperatures for the different observations, is poured into the box. At each of these temperatures, the deflexion produced on B is observed, and the thermometer is read. Thus we obtain the values of $\frac{A}{E}$ at different temperatures of A; and, as the experiment can be completed in so short a time that we may presume on the invariability of E during the observations, we find in fact the proportionate change of A for a given change of temperature.

In some instances, the temperature has been changed by heating the air of the room.

The ratio of change thus found is very different for different magnets. It probably depends on the quality of the steel; or possibly on the mode of magnetization: but on this point nothing is known with certainty. In all instances, it is believed, the magnet-power diminishes when the temperature is raised. In the "Horizontal Force Magnet" of the Royal Observatory, Greenwich, the loss of power for a rise of 1° Fahrenheit is = magnet-power $\times 0{\cdot}0000809$: in the magnet lately used as "Vertical Force Magnet," the loss of power for 1° is = magnet-power $\times$ 0·00013845.

In using a needle in the manner described in Article 25, for comparing the terrestrial horizontal force at different localities, the result ought always to

be reduced, by applying the correction for temperature, to what it would have been at a fixed temperature of the magnet, as 32°.

32. *Accurate determination of the Earth's local horizontal magnet-power, founded on the method of deflexions, used in combination with the measure of the Earth's horizontal action upon the disturbing magnet.*

In the experiment of Article 26, or that of Article 27, treated in the manner described in Article 29, we have found $A = k \times E : k$ being a number determined by the solution of certain equations given by the observations. In those observations it was necessary to use a needle B: but we have no further occasion to employ that needle. The operation now to be performed consists in suspending the magnet A delicately, and observing the time of its vibration under the action of the Earth's horizontal force. The theory of this has been treated in Article 25; and the result, substituting A for B, and preserving T to denote the time of a complete double vibration, and M for the moment of inertia of A, is

$$T = 2\pi \sqrt{\frac{M}{EA}} :$$

$$\text{whence} \qquad EA = \frac{4\pi^2}{T^2} \times M.$$

It will be remarked that the unit, by which E and A are measured, is the same as that described at the end of Article 29, namely, the action of the standard magnet S upon S' at distance 1.

Multiplying together the two equations

$$\frac{E}{A}=\frac{1}{k}, \qquad EA=\frac{4\pi^2}{T^2}\times M,$$

and extracting the square root,

$$E=\frac{2\pi}{T}\sqrt{\frac{M}{k}}:$$

a result which depends upon no property whatever of the magnets employed, except M the moment of inertia of A. To the determination of the numerical value of M we will now give our attention.

If, as is usually the case with large magnets, the form of the magnet be a parallelopiped, and its structure homogeneous, the value of M will be $\frac{\text{mass in grains}}{12}\times$ {(length in feet)^2 + (breadth in feet)^2}. This calculation is easily made with accuracy. It is necessary to add the moment of inertia of the stirrup or other hook which supports the magnet and vibrates with it: this in general is a very small quantity, and can be obtained with sufficient accuracy by weighing that apparatus and estimating its radius of gyration.

If the form of the magnet be not so simple, or if there be any grounds for suspicion of the accuracy of this process, the device proposed by Weber may be adopted. The time of vibration of the magnet having been observed, as above mentioned, the two ends of the magnet are then loaded with brass weights, very carefully weighed, which rest upon the magnet by sharp points, so that the weights do not partake of the cir-

cular movement of the magnet; and the distance between these points is measured. In that state, the time of vibration is again observed. As the force which causes the vibration is the same in both cases (namely, the action of terrestrial horizontal magnetism upon the magnet), the moments of inertia will be proportional to the squares of the two times of vibration. But the difference between the two moments of inertia is merely the moment of inertia of the two brass weights, each being supposed collected at its sharp supporting-point, and admits of being accurately computed. Then the difference of the moments of inertia of the magnet in its two states being known, and their proportion being known, each of them is determined accurately.

Whichever method is used, the numerical value of M is found and substituted in the expression for E, and the numerical value of E is obtained.

In the observation of deflexion described in Articles 25 to 29, it is evident that the comparison of the magnet-powers E and A implies that their numerical values are referred to the same unit. And in the investigation, in the present Article, of the measure of the Earth's action upon the magnet, we have used exactly the same formula as in Article 21, which is founded on the methods of preceding Articles, in which all are referred to the same unit. It follows that the numerical value which we have found for E is referred to the same unit: namely (see Article 29) to the magnet-power, or to the magnetic action at distance 1, of the standard magnet S or its equal S', which are such that, when

the distance of their centers is 1, the angular momentum produced in S' by S broadside-on (using the first term only of the formula of force), is 1.

In general, the two operations (deflexion and vibration) can be performed in so short a time that the effects of change of temperature and change in the Earth's force may be neglected. If it is thought necessary to recognise them, reference must be made, for temperature-correction, to the experiments of Article 31, and for the terrestrial change, to observations of minute changes (to be mentioned hereafter, Article 83.) Corrections for torsion-force of the suspension-thread ought to be applied on the principles of Article 25. There is also another small correction, for magnetism *induced* in the needle A by the Earth's action: for this we must refer to a succeeding section, Article 72.

33. *Investigation of the proportion in which the numerical value for* E *will be altered, when, instead of using the foot and the grain as units, we use other units, as the millimètre and milligramme.*

Let the foot $= p$ millimètres, and the grain $= q$ milligrammes. Suppose the observations of Articles 28 and 31 adopted without any modification of circumstances, and let us examine how the resulting formulæ will be modified by use of the new units. The experiment of Article 27 practically gives $2A.c^{-3} = E.\sin\phi$; or $\frac{E}{A} = \frac{2}{\sin\phi}c^{-3}$. With the new units, $\sin\phi$ will not be altered, but c (numerically) will be p times

as great as before, and $\frac{E}{A}$ will be (numerically) p^{-3} times as great as before. The experiment of Article 32 gives $EA = \frac{4\pi^2}{T^2} M$; T is not altered by the new units; but M, which depends on product of mass by square of distance from center of angular motion, will be (numerically) $p^2 q$ times as great as before. The product of $\frac{E}{A}$ by EA will therefore be (numerically) $\frac{q}{p}$ times as great as before; and the numerical expression will be $\sqrt{\frac{q}{p}}$ times as great as before.

The value of p is 304·7947: that of q is 64·79895; therefore the new numerical expression for E on the Metric system will be formed by multiplying that on the English system by $\sqrt{\frac{64{\cdot}79895}{304{\cdot}7947}}$ or 0·461084.

The same numerical result would have been obtained if the units employed, in the Metric system, had been the mètre and gramme.

34. *Special values of* E: *historical physical change in the value: lines on the Earth's surface passing through points of equal horizontal force.*

The mean value of E found at Greenwich in the year 1867 was 3·851 in English measure, or 1·776 in Metric measure. In 1848 its value at Greenwich, in English measure, was 3·722. The increase in 19 years

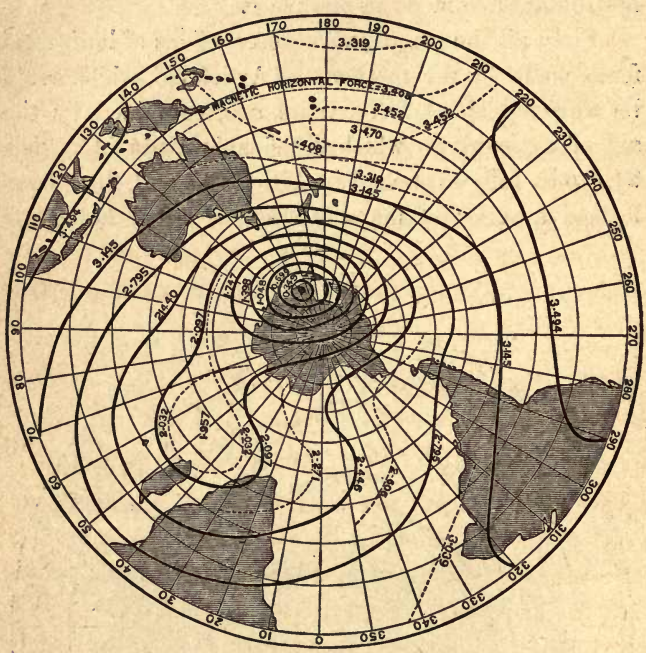
is in the proportion of 29 to 30: its rate of increase in successive years is sensibly uniform. We believe that this is the longest series of accurate determinations of horizontal force made in one place.

From all the comparative observations of horizontal force, made by the methods of Article 25, which could be collected about forty years ago, combined by the aid of a theory to which allusion is made in Article 24, (to be fully explained in Articles 47 and 49), Gauss formed a series of maps of lines of equal horizontal

Fig. 28.

magnetic intensity, which are copied, without essential change, in Figures 28 and 29. These maps are on the

Fig. 29.



stereographic projection. The numbers upon the lines give the value of E in Metric measure. Remarks on the peculiarities of these curves will be given below, in Article 42.

SECTION V.

On Terrestrial Magnetism, as acting in the vertical at each place of observation; and on the combination of the horizontal and vertical forces, and the total terrestrial magnetic force at each place of observation.

35. *First evidence of the existence of a vertical magnetic force.*

When a needle is prepared, in the unmagnetized state, for mounting in a compass, with its center of gravity very little below its point of support, and is adjusted to horizontality; on being magnetized, its red end (in northern latitudes) dips considerably. This proves that (in northern latitudes) the terrestrial horizontal magnetic force towards the north is accompanied with a vertical force downwards, and the terrestrial horizontal force towards the south is accompanied with a vertical force upwards.

When the same compass is carried into southern latitudes, the blue end dips. This proves that, while the sign of the terrestrial horizontal force in the north direction or in the south direction has not changed, the

sign of the vertical force has changed. This is so well known that, in the best compasses, a sliding weight is provided, which in north latitudes can be applied to the blue end of the needle, and in south latitudes can be applied to the red end of the needle.

The instrument with which this vertical force is most conspicuously exhibited and most accurately examined will be described in the next article.

36. *Description of the Dipping Needle.*

The function of this instrument is limited strictly to the determination of the direction which a needle will take under the action of the total terrestrial magnetic force, when it is constrained to move in an arbitrary vertical plane. This limitation permits the construction of an instrument possessing great simplicity, and, in consequence (viewing the nature of its action) great accuracy.

The needle must be carried by a horizontal axis, passing as nearly as practicable through its center of gravity. This condition, though convenient, is not necessary: for, as will be shewn in the next Article, we can so arrange the observations as perfectly to eliminate the effects of error of position of the axis: and indeed, for some observations, the place of the center of gravity is purposely moved to a sensible distance from the axis. The axis must terminate in two delicate pivots; and it is mainly in the formation of these that the utmost skill of the artist is required. It is very difficult so to arrange the observations that the injurious effect of an

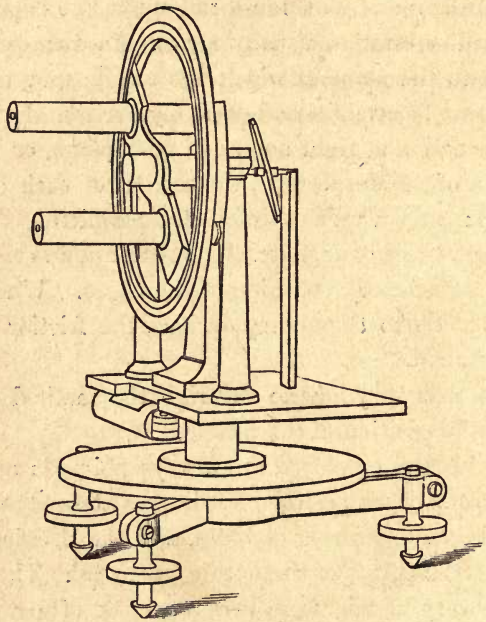
oval or otherwise ill-formed pivot can be entirely removed. To make these pivots turn with the least possible friction is of the utmost importance: and for this object, the pivots must not turn in Ys like those of a transit-instrument, but must roll upon two edges of a very hard substance, usually agate. In the direction parallel to the plane in which the needle moves, these edges must be straight and perfectly horizontal; in the vertical section at right angles to that plane, or in the direction of the needle-axis, the section of each edge is rounded; a form very desirable for permitting the escape of particles of dust, &c. Great attention is required for the satisfactory polishing of the edges. When due care is given to these preparations, the friction is extremely small.

It is necessary now to describe the method of observing the position of the needle.

The needles employed are always pointed: and, till within a few years past, the needle was allowed to swing within a graduated ring of brass, and the divisions opposite to the points of the needle were read. The reading was very rough, and there was risk of error from the close proximity of the needle to the brass, which is seldom perfectly free from iron. Lately, a far superior form has been introduced, known as the Kew pattern (from the circumstance that it was invented and first used at the Kew Observatory). A view of that instrument is given in Figure 30. Several auxiliary parts, unimportant to the general principle, are omitted in this drawing. There is no metal near the needle: the

points of the needle are observed by means of microscopes which are attached to a revolving frame that carries verniers by which the graduations of an external

Fig. 30.



vertical circle are read. The true position of the needle (including all effects of friction, uncertainty of reading, &c.) is rarely doubtful to the extent of 2′. A modified form of the instrument, adapted to the use of needles of different lengths, and with other fittings, is mounted as a permanent instrument at the Royal Observatory, Greenwich.

In all cases, the instrument is so mounted that it

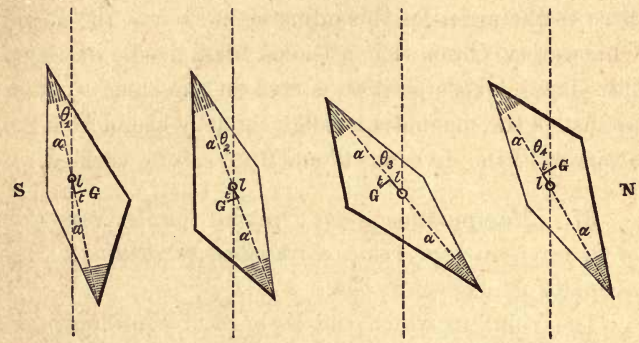
can rotate round an axis, which must be made accurately vertical. The divisions 0° and 180° of the circle as read by the microscopes ought then to correspond to vertical position of the needle. In general, it is sufficient to trust to the artist for this adjustment: but at the Royal Observatory, Greenwich, a loaded brass needle has been introduced, whose position is read in the same manner as that of the magnetic needles; and by use of this the accuracy of the divisions 0° and 180° can be verified.

37. *Manipulation of the Dipping-Needle: reversion of its pivot-bearing: rotation round its vertical axis: reversal of its magnetic poles.*

The point to which the theoretical considerations, employed in the use of the dipping-needle, are particularly addressed, is the elimination of errors produced by the non-coincidence of the axis of rotation of the needle with its center of gravity. Measuring from the axis of rotation, the center of gravity may have an ordinate of sensible value in the direction longitudinal to the needle, and one in the direction transversal to the needle. It will be easily conceived that the effect of the latter ordinate may be eliminated by reversing the pivots upon the agate edges, so as to present that edge upwards which was downwards (see the difference between Figures 31 and 32, or between Figures 33 and 34). The same effect may be produced by rotating the entire supporting frame with the needle which it carries, round a vertical axis, through 180°: for, as the vertical force of magnetism tends to depress the same end as before,

while the horizontal force, always drawing that end to the magnetic north, now draws it to a part of the supporting frame opposite to the former, the edge of

Fig. 31. Fig. 32. Fig. 33. Fig. 34.



the needle which was below will now be above. It is evident here that we shall have the means of eliminating the effect of that ordinate of the centre of gravity which is transversal to the needle.

But for eliminating the effect of that ordinate which is longitudinal to the needle, we must have some method of altering the direction of magnetism with respect to that ordinate. And there is but one way of doing this; namely, by reversing the poles of the magnet. This, which may be done by the power of a galvanic current, or in other ways, is done in practice most conveniently by the method of double-touch, described in Article 8. Inasmuch as the application of a pair of magnets will produce certain magnetism in a needle, it is easily conceivable, and is accurate in fact, that the use of the opposite ends of those magnets (which possess magnet-

ism of the kind opposite to that of the ends first used) will first destroy the magnetism planted in the needle, and will then plant in it new magnetism of the opposite kind. It is only necessary to caution the operator that the touch-magnets used must have much greater magnetic power than the needle: otherwise it might happen that the needle, fully charged with magnetic power, would reverse the poles of the touch-magnets. With touch-magnets of adequate power, this never happens: the poles of the needle are reversed, without injury to the powers of these magnets: and the magnetic power of the needle in its state of reversed magnetism is sensibly equal to that before reversion, as is ascertained by calculations to be mentioned below.

38. *Mathematical theory of the Dipping-Needle: first, on the supposition that the magnetic intensity after reversion is equal to that before reversion; simplification when the needle is very nearly balanced.*

In Figures 31, 32, 33, 34, the same part of the edge of the dipping-needle is represented by the strong line in a portion of the outline. The magnetic north is supposed to be to the right. Commencing with Figure 31, the needle is so turned in Figure 32 that the edge which was downwards is now upwards, but no change is made in the magnetism of the needle. After this, the magnetism is reversed; and, as is seen in the shading of the Figure, the end, which was charged with blue magnetism and was uppermost, is now charged with red magnetism and is lowest. Between Figures 33 and 34

the needle is so turned that the edge of the needle is reversed in regard to up and down. G is the place of the center of gravity, whose longitudinal and transversal ordinates will be called l and t. Put W for the weight of the needle. Let H and V be the horizontal and vertical terrestrial magnetic forces which act on each pole of the needle before reversion of its magnetism: pulling the red or lower pole, horizontally in the plane of vibration, and downwards, respectively; and pulling the blue or upper end in the opposite directions: and let nH and nV be the similar forces after the reversion of the poles. (This amounts to the same as supposing that the magnetic intensity after reversion is to that before reversion as n to 1.) Let a be the distance of each of the poles from the center. Our object now is to find the proportion of H to V.

In Figures 31, 32, 33, 34, let θ_1, θ_2, θ_3, θ_4, be the angles made with the horizon by the line joining the poles of the needles, which angle in each case may be called the apparent dip.

Then in Figure 31 the equation of equilibrium will be

$$W \times (l \cos \theta_1 + t \sin \theta_1) + 2Va \cos \theta_1 - 2Ha \sin \theta_1 = 0;$$

or $$W \times (l \cot \theta_1 + t) + 2Va \cot \theta_1 - 2Ha = 0.$$

Similarly,

in Figure 32, $W \times (l \cot \theta_2 - t) + 2Va \cot \theta_2 - 2Ha = 0$;

in Figure 33, $W \times (-l \cot \theta_3 - t) + 2nVa \cot \theta_3 - 2nHa = 0$;

in Figure 34, $W \times (-l \cot \theta_4 + t) + 2nVa \cot \theta_4 - 2nHa = 0$.

The simplest case of these equations will be that given by the usual assumption, that the intensity of magnetism is the same after reversion, or that $n=1$. The four equations then become

$$W\times(l\cot\theta_1+t)+2Va\cot\theta_1-2Ha=0;$$

$$W\times(l\cot\theta_2-t)+2Va\cot\theta_2-2Ha=0;$$

$$W\times(-l\cot\theta_3-t)+2Va\cot\theta_3-2Ha=0;$$

$$W\times(-l\cot\theta_4+t)+2Va\cot\theta_4-2Ha=0.$$

Adding the first pair, and dividing by $\cot\theta_1+\cot\theta_2$,

$$Wl+2Va-\frac{4Ha}{\cot\theta_1+\cot\theta_2}=0.$$

Adding the second pair, and dividing by $\cot\theta_3+\cot\theta_4$,

$$-Wl+2Va-\frac{4Ha}{\cot\theta_3+\cot\theta_4}=0.$$

Adding these equations,

$$4Va-4Ha\left\{\frac{1}{\cot\theta_1+\cot\theta_2}+\frac{1}{\cot\theta_3+\cot\theta_4}\right\}=0.$$

Now, considering the vertical force V and the horizontal force H as resolved parts of the total terrestrial force acting on the needle, it is seen that $\frac{V}{H}$ is the trigonometrical tangent of the angle which the direction of the total force makes with the horizon, or is the tan. Dip. Thus we obtain

$$\text{tan. Dip}=\frac{1}{\cot\theta_1+\cot\theta_2}+\frac{1}{\cot\theta_3+\cot\theta_4}.$$

If the needle is very nearly balanced on its pivots (a condition which the artist always endeavours to secure), so that the four angles θ_1, θ_2, θ_3, θ_4, are nearly equal: then we have, without perceptible error,

$$\cot\theta_1+\cot\theta_2=2\cot\frac{\theta_1+\theta_2}{2};\ \cot\theta_3+\cot\theta_4=2\cot\frac{\theta_3+\theta_4}{2};$$

$$\text{and tan.Dip}=\frac{1}{2}\left\{\tan\frac{\theta_1+\theta_2}{2}+\tan\frac{\theta_3+\theta_4}{2}\right\}$$

$$=\tan\frac{\theta_1+\theta_2+\theta_3+\theta_4}{4}$$

(by the same reasoning as that just employed): and consequently

$$\text{Dip}=\frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4).$$

39. *Mathematical theory of the Dipping-Needle continued: secondly, on the supposition that the intensity is not the same after reversion, and that the needle is not nearly balanced.*

We must now use the accurate equations near the beginning of the last Article: and first, to find the value of n.

Multiply the first equation by $\tan\theta_1$ and the second by $\tan\theta_2$, and subtract;

$$Wt\,(\tan\theta_1+\tan\theta_2)-2Ha\,(\tan\theta_1-\tan\theta_2)=0.$$

Multiply the third equation by $\tan\theta_3$ and the fourth by $\tan\theta_4$ and subtract;

$$Wt\,(\tan\theta_4+\tan\theta_3)-2nHa\,(\tan\theta_4-\tan\theta_3)=0.$$

Eliminating at the same time Wt and Ha,

$$n = \frac{(\tan\theta_4 + \tan\theta_3)(\tan\theta_1 - \tan\theta_2)}{(\tan\theta_4 - \tan\theta_3)(\tan\theta_1 + \tan\theta_2)}$$

$$\text{or} = \frac{(\cot\theta_3 + \cot\theta_4)(\cot\theta_2 - \cot\theta_1)}{(\cot\theta_3 - \cot\theta_4)(\cot\theta_2 + \cot\theta_1)}.$$

This expression can be used with accuracy when the needle is greatly out of balance, so that θ_1 and θ_2 differ considerably and θ_3 and θ_4 differ considerably: but it is not accurate when these angles approach to equality, because the unavoidable errors of observation then greatly affect the proportionate accuracy of $\cot\theta_2 - \cot\theta_1$ and of $\cot\theta_3 - \cot\theta_4$.

On expanding the numerator and denominator, it will be found that the supposition $n = 1$ corresponds to this very simple equation:

$$\tan\theta_1 . \tan\theta_3 = \tan\theta_2 . \tan\theta_4;$$

$$\text{or } \cot\theta_1 . \cot\theta_3 = \cot\theta_2 . \cot\theta_4.$$

But this equation, which is founded on the last, cannot be used with safety when the needle is very nearly balanced.

From the first pair of the original equations, and from the second pair, we find

$$Wl + 2Va - \frac{4Ha}{\cot\theta_1 + \cot\theta_2} = 0,$$

$$-Wl + 2nVa - \frac{4nHa}{\cot\theta_3 + \cot\theta_4} = 0;$$

of which the sum is

$$(2+2n)\,Va - 4Ha\left(\frac{1}{\cot\theta_1+\cot\theta_2}+\frac{n}{\cot\theta_3+\cot\theta_4}\right)=0;$$

whence

$$\text{tan. Dip} = \frac{V}{H} = \frac{2}{1+n}\left(\frac{1}{\cot\theta_1+\cot\theta_2}+\frac{n}{\cot\theta_3+\cot\theta_4}\right).$$

If n has been numerically calculated, from the formula above, the computation of tan . Dip may most readily be made by substituting the numerical value in this expression. Or, if the symbolical expression for n be substituted,

$$\text{tan. Dip} = \frac{2(\cot\theta_2-\cot\theta_1+\cot\theta_3-\cot\theta_4)}{(\cot\theta_2+\cot\theta_1)(\cot\theta_3-\cot\theta_4)+(\cot\theta_2-\cot\theta_1)(\cot\theta_3+\cot\theta_4)}.$$

In ordinary observations with the dipping-needle, these formulæ are not required. But they are required in the following case. The adjustment of the instrument which it is peculiarly beyond the power of the observer to verify, is the circularity of the pivots. Some observers therefore have thought it desirable that the needle should be so unequally loaded as to be sensibly out of balance, thus making the apparent dips θ_1, θ_2, θ_3, θ_4 very unequal, and bringing different sides of the pivot into bearing upon its agate edges. With that arrangement, the formulæ above investigated are necessary.

40. *Theory of the Dipping-Needle when the dips are observed in different vertical planes inclined to the plane of the magnetic meridian.*

The investigations above apply to the case of the needle vibrating in the magnetic meridian: and they then give for result the true magnetic dip. It is only necessary that the direction of the magnetic meridian be nearly known: a small error is unimportant, and the determination of the meridian by a common compass is abundantly accurate for this purpose.

But the investigations also apply when the needle vibrates in any other vertical plane. They do not then, however, give the true dip; they give only an apparent dip, corresponding to the proportion of the vertical magnetic force to the resolved part of the horizontal magnetic force in that plane. (For it is obvious that the part of the horizontal magnetic force which is perpendicular to that plane, being parallel to the needle's axis of rotation, can have no effect on its rotation or vibration.) If ϕ_1 and ϕ_2 be the azimuths from the magnetic meridian, measured in the same direction, of two vertical planes in which the dip is observed; d_1 and d_2 the apparent dips observed in those planes: then the resolved horizontal forces in these planes will be $H \cos \phi_1$ and $H \cos \phi_2$, and the equations between the apparent dips and the azimuths will be,

$$\text{cotan. } d_1 = \frac{H \,.\, \cos \phi_1}{V}; \quad \text{cotan. } d_2 = \frac{H \,.\, \cos \phi_2}{V}.$$

But $\frac{H}{V} = \text{cotan. True Dip}$; therefore

$$\text{cotan. } d_1 = \text{cotan. True Dip} \times \cos \phi_1;$$

$$\text{cotan. } d_2 = \text{cotan. True Dip} \times \cos \phi_2.$$

There is only one case of these equations which offers any interest. If $\phi_2 = \phi_1 + 90^\circ$, then $\cos \phi_2 = -\sin \phi_1$; and the sum of the squares of the two equations becomes

$$(\text{cotan. } d_1)^2 + (\text{cotan. } d_2)^2 = (\text{cotan. True Dip})^2,$$

from which the angle ϕ_1 has disappeared.

Thus it appears that the True Dip can be obtained from observation of the apparent dips in two planes, with no condition as to the position of these planes except that their azimuths differ by 90°. This condition can always be secured by means of the azimuthal circle on which the dip-apparatus is mounted.

41. *Determination of the Total Terrestrial Magnetic Force at any locality: lines upon the Earth's surface passing through points of equal dip, and lines passing through points of equal Total Force: historical changes.*

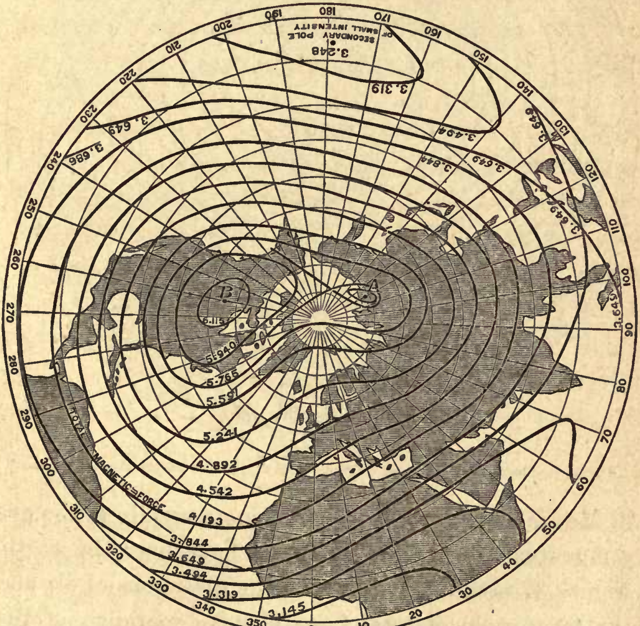
By the investigations extending from Articles 26 to 34, the terrestrial horizontal magnetic force is measured. And by those from Articles 38 to 40, the dip is measured. It is plain that the Total Force = Horizontal Force × secant of True Dip: and thus the total terrestrial horizontal magnetic force is ascertained, without risk of inaccuracy, except at points where the dip is nearly vertical (that is, at points near the magnetic poles).

We confine our attention to this method, because it is the only one which does not rely on the constancy of a needle's magnetism, and because it is very accurate except close to the magnetic poles, where the value of the total force can be inferred from those around it by the laws of continuity. Determinations have been made,

however, by observing the time of vibration of a dipping-needle in the magnetic meridian: or by observing the extent to which the the needle is displaced by a given weight attached to a thread which is wrapped round the axis of the needle. The theory of these is so simple that there is no need to delay on them.

The ring-shaped lines in Figures 20 and 21 represent

Fig. 35.

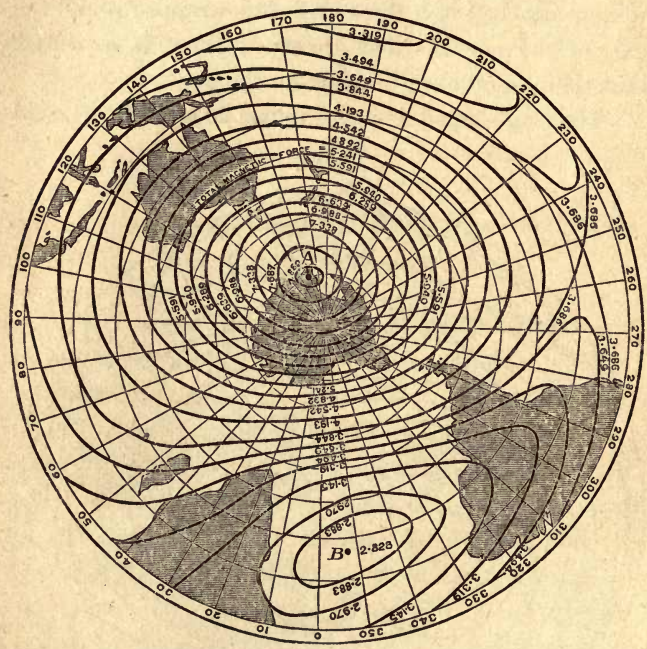


(B is the primary and A the secondary northern pole of greatest magnetic intensity.)

the lines of equal dip over the surface of the earth: and the lines in Figures 35 and 36 represent the lines of

equal total magnetic force. The numbers upon the latter system of lines shew the value of the total force

Fig. 36.

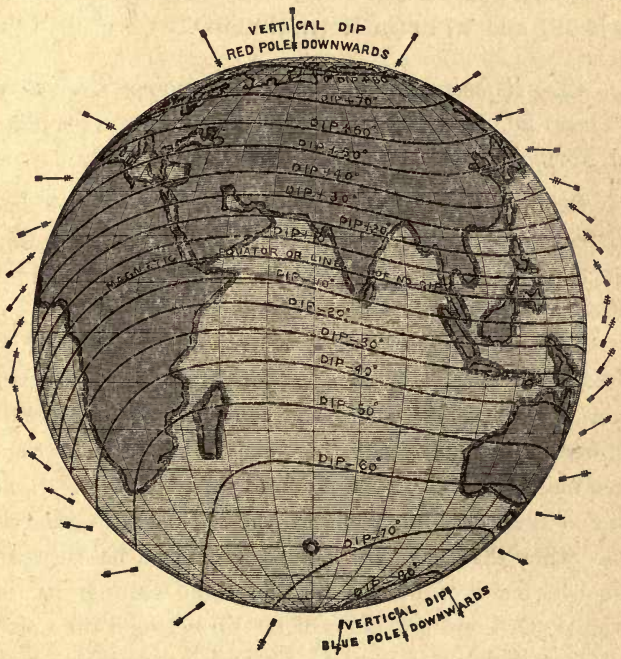


(A is the southern pole of greatest magnetic intensity, and B the primary pole of small intensity.)

in Metric measure. To present more vividly to the eye the general facts of dip and total force over the earth, Figure 37 is drawn, exhibiting the directions of dip and the magnitude of total force along a meridian of the earth. The magnitude of force is shewn rudely by the lengths of the symbolical needles at the different points of the meridian. The map is on the orthographic pro-

jection. It will be remarked that there is a little inaccuracy near the south pole, arising from the circum-

Fig. 37.



stance that it is impossible to include the north and the south magnetic poles in the same geographical meridian.

The Dip and the Total Terrestrial Magnetic Force at any place, like the elements of which we have treated in Articles 24 and 34, are slowly changing. In 1843 the dip at Greenwich was about 69° 1′; it has diminished, with a rate continually accelerating, till in 1868 it

was 67° 56′. Adopting as elements of calculation that in 1848 the dip and horizontal force were 68° 47′ and 3·76, and, in 1866, 68° 1′ and 3·85: the total force was, in 1848, 10·39, and, in 1866, 10·28 (in English units), or 4·791 and 4·740 (in Metrical units).

42. *Reference to the points of principal interest in Figures* 20, 21, 28, 29, 35, 36*: secular change in the place of North Magnetic Pole.*

Before entering upon the consideration of the diagrams, we will allude to some general points regarding the connexion of the magnetic meridians with the curves of equal dip and of equal horizontal force.

Adopting, as the most convenient definition of "Magnetic Pole," (when not qualified by any other words), "the point where the dip is vertical," there is no reason in nature why there should not be more than one magnetic pole in the north (or in the south). If, at one of these proximate places, the red pole dips, and at the other the blue pole dips, there must be between them a place of no dip (in the same manner as, in Figure 37, there is a place of no dip between the north and south poles of vertical dip with opposite poles of the needle). But, if the red pole dips at both, there is some complication introduced into the forms of the equal-dip curves. We will however commend these to the examination of the speculative student: remarking that we have no reason to think that there is more than one Magnetic Pole or place of vertical dip either in the north or in the south.

Now in progressing along one of the magnetic meridians in Figure 20, the observer who follows the direction of the horizontal needle is in fact continually proceeding in the plane of dip. And, if he finds the dip continually increasing (that is, if he advances towards the smaller circles of Figure 20), he will at last arrive at the place of vertical dip. Or, conversely, if he starts from the place of vertical dip, and continues in the course defined by any one of the directions of the horizontal needle into which he will immediately fall, he will pass away from the place of vertical dip in the plane of dip, and will therefore, for a time at least, have dip continually diminishing. From all this it appears that the pole of no dip must be the same as the pole common to every magnetic meridian; that is, the pole to which all magnetic meridians converge.

The pole to which the lines of equal horizontal force are related, that is, the point where horizontal force vanishes, is evidently the pole of vertical dip.

Thus the Magnetic Pole is a common pole for the convergence of magnetic meridians, for the verticality of dip, and for the evanescence of horizontal force.

But the pole of greatest total force is entirely different in its properties from these. It has not necessarily any connexion with them. There may be any number of points where the total force is maximum (in comparison with the points that surround them, to a considerable distance). The number of such points in the north may be different from that in the south.

We will now proceed with the diagrams. And first

it must be stated that these diagrams are not, and could not be, drawn simply from observations. They are drawn from a theory (to be explained hereafter) founded upon all the observations which could be collected before the preparation of the charts (published in 1840). In general they represent very accurately the facts of observation: but in later years some sensible but not important inaccuracies have been discovered in the southern hemisphere.

In Figures 20 and 21, it is to be remarked that the magnetic meridians might have been drawn through any arbitrary points of the geographical equator; they are in fact drawn through the points of east longitude 8°, 18°, 28°, &c.: 8° and 188° being the points at which the Magnetic Equator or line of no dip crosses the geographical equator. The magnetic meridians cannot generally be great circles of the sphere, because the two Magnetic Poles through which all must pass are not exactly opposite: they have moreover other irregularities of form, which do not depend on the character of the stereographic projection, but are equally conspicuous when the curves are traced on a globe. There is no trace of more than one pole either in the north or south.

The form of the lines of equal dip is remarkable. Commencing with the line of no dip or Magnetic Equator, it is easily seen that it is not a great circle: its greatest northerly distance from the geographical equator occurs at about 55° east longitude, or 47° from its node (instead of 90°), and its greatest southerly

distance at about 318° east longitude, or 50° from the same node. The inclination of the Magnetic Equator to the geographical equator is greater near the west coast of Africa than in any other part. Proceeding to the neighbouring lines, it will be seen that the increase of dip is nearly double of the increase of latitude; and upon this circumstance was founded the conjectural law, tan. Dip = 2 tan. distance from magnetic equator; to which we shall advert in the next Section. Nearer to the Magnetic Poles the curves are oval, or rather pear-shaped; but the major axes of the northern curves, and those of the southern curves, are not in the same direction. The North Magnetic Pole (in longitude 265°) and the South Magnetic Pole (in longitude 152°) are not opposite each other. These remarks show that the Earth's magnetism cannot be represented as the power of one magnet, and that the distribution of magnetism about the Earth is unsymmetrical.

The curves of equal Horizontal Force, Figures 28 and 29, are still more strange. The greatest values are 3·733 at the geographical equator in longitude 259°, and 3·673 in 14° north latitude, longitude 103°. Proceeding along an equatoreal belt, one minimum of 3·039 is reached in longitude 345°, north latitude 3°, and another 3·408, in longitude 156°, south latitude 13°. Proceeding in either direction, north or south, from this equatoreal belt, the Horizontal Force gradually diminishes to 0 at each pole. The north pole is in the north of Baffin's Bay: the south in South Victoria.

But the forms of the southern curves only seem to indicate the existence of two poles of magnetic force. This indication differs remarkably from that which is founded upon the system of curves to be mentioned next.

The curves of equal Total Magnetic Force present us with the singular phenomenon of two poles of maximum force in the north, and only one in the south. The numerical values of the forces at the former are, 6·160 west of Hudson's Bay, and 5·911 in Siberia: that of the latter 7·898 in South Victoria. Proceeding from these towards the equatoreal belt, the equatoreal maxima 3·686 and 3·649 are reached in longitude 252°, south latitude 7°, and in longitude 110°, north latitude 6°, and the equatoreal minima 2·828 and 3·248, near St. Helena in longitude 355°, latitude 16° south, and in longitude 179°, latitude 6° north. There is a rude approach to the law, that the Total Force at the Magnetic Poles is double that at the Magnetic Equator.

The theoretical connexion of these facts will be treated in the next Section.

At the end of Article 41 it was remarked that, at Greenwich, the Dip and Total Force are diminishing. Interpreting these by the remarks above, it would seem that the Magnetic Equator is approaching to Greenwich, or the North Magnetic Pole is receding from Greenwich. And remarking also the westerly change in direction of north magnetic meridian, from the sixteenth century to the year 1824, and its subsequent easterly motion (Article 24), it would seem that

the north magnetic pole has rotated round the terrestrial pole in a small circle from east to west, and having passed the point where its westerly azimuth as viewed from Greenwich is maximum, it is still continuing its course in that circle. It seems probable that in the fifteenth or sixteenth century it was situated between North Cape and Spitzbergen: it is now north-west of Hudson's Bay.

Valuable information on these changes, from the earliest period to the years about 1830, will be found in the work "Terrestrial and Cosmical Magnetism, the Adams Prize Essay for 1865, by Edward Walker, M.A." (Deightons, Cambridge). There is also some accurate information applying to later years, but not possessing all the completeness which might have been obtained from published records.

Section VI.

Theories on the Physical Cause or Representation of Terrestrial Magnetism.

43. *Reasons for believing that Terrestrial Magnetism is not produced, in any important degree, by magnetic forces external to the earth.*

If there were an external cause for magnetism, it seems scarcely conceivable that some large part of it would not act in planes parallel to the geographical equator: and, if so, its effects at any one place would undergo very great changes in the earth's diurnal revolution; every part of the earth being presented, in the course of a day, in different aspects towards forces so acting. Now the fact is that the diurnal changes are very small, perhaps at Greenwich $\frac{1}{500}$ part of the whole horizontal force. It would seem therefore certain that external bodies or space do not produce any sensible part of the magnetism in the planes to which the earth's axis is normal. And this carries with it a very strong improbability that they produce any sensible magnetic forces in the direction of the earth's axis also.

44. *Reasons for believing that Terrestrial Magnetism does not reside, in any important degree, in the earth's surface.*

The first class of reasons are those general ones which are founded on ordinary observation, of the materials of which the earth's surface is composed, and of their non-magnetic property: and upon the general absence of any perceptible change in magnetism depending on the change of soil. The materials of a clay-field are not sensibly magnetic, nor are those of a sand-field, nor is there any change of the general terrestrial magnetism in going from one to the other; nor are the granite rocks in one district, or the limestone rocks in another, sensibly magnetic. In some places there are ferruginous rocks, specimens of which when brought near to a delicate compass are found to produce sensible disturbance: but the great masses of those rocks on the earth's surface, when examined (by examination of the declination, dip, and horizontal intensity) at corresponding distances in their neighbourhood, produce no sensible disturbance.

The second class of reasons consists of those founded on measures of the magnetic elements at different elevations above the earth's surface. One series includes the observations taken on mountain-heights: of these the most valuable are those of Professor James Forbes (Edinburgh Transactions, vol. xiv.), from which it appears that, for a height of 100 feet, horizontal magnetic force is diminished, in Europe, by $\frac{1}{30000}$ part,

and dip is increased by 5″. Both these would correspond to the supposition that the magnetic power is sensibly below the earth's surface. As the observer was not actually separated from the earth, the validity of inference from these may be disputed. Another series is that of observations in balloons, which are free from every objection of that kind, but which are not quite so accurate; and which are necessarily almost limited to observations of horizontal intensity, as found by vibrations (Article 25). The following are the results of these observations :—

Gay Lussac, 1803, at the height 4000 metres found no sensible diminution of magnetic force (*Annales de Chimie*, vol. 52).

Gay Lussac, 1804, at the height 6900 metres found an apparent very small increase: but this was probably caused by the low temperature of the needle, for which no correction was applied. The dip, imperfectly observed, was not sensibly altered. (*A. de C.* vol. 52.)

Glaisher, 1862, found at the height 20200 feet a diminution of power: but in other observations at 5300, 11000, and 3800 feet, found the same as on the earth (*Report of British Association* 1862).

Glaisher, 1864, found a diminution of about $\frac{1}{7}$ part at the height 14000 feet. (*R. of B. A.* 1864.)

Glaisher and Evans, 1864, found an even larger diminution at height 3600 to 5000 feet. (*R. of B. A.* 1865.)

It would appear generally from these observations, that there is a sensible diminution of magnetic hori-

zontal force at a great elevation. But the last set of observations casts much doubt on this conclusion. It is to be remarked that all the balloon-observations at great height were compared with observations on the earth. It might have been safer to compare them with balloon-observations at small elevations. Now the last set of observations seems to shew that an apparent large diminution arises simply from the effect of localization in the balloon-car; and, if this be accepted, there is scarcely any sensible effect to be ascribed to the great elevations.

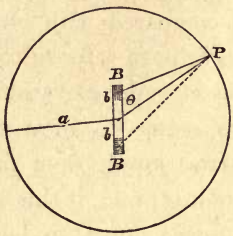
Now, remarking how rapidly magnetic power diminishes with increase of proportion of distance from the magnetic poles, it follows from the observations above that the height of three or four miles must bear a small proportion to the distance of the magnet which produces the magnetic power observed at the earth's surface, and therefore the source of magnetism must be deep.

45. *Attempt to explain Terrestrial Magnetism by the action of a magnet of small dimensions but of very great power, near the center of the earth.*

About the middle of the last century it was suggested by Mayer, and in the present century the same idea was independently adopted by Humboldt and Biot (Biot, *Traité de Physique*, 1816, vol. iii. page 139), that the principal phænomena of Terrestrial Magnetism could be explained by the action of a powerful magnet, of limited dimensions, near the center of the earth. Its

theory is as follows. In Figure 38, let the magnetic

Fig. 38.



pole be defined by prolonging the axis of the magnet till it cuts the earth's surface; θ will be the complement of magnetic latitude. The action of the northern or blue pole upon the red end of a needle at P will be represented by

$$B\ (a^2 + b^2 - 2ab\ \cos\ \theta)^{-1}:$$

its resolved part in the horizontal plane at P, towards the pole, will be

$$Bb\ .\ \sin\ \theta\ .\ (a^2 + b^2 - 2ab\ \cos\ \theta)^{-\frac{3}{2}};$$

the action of the red pole in the same direction will be

$$Bb\ .\ \sin\ \theta\ .\ (a^2 + b^2 + 2ab\ \cos\ \theta)^{-\frac{3}{2}}:$$

the total horizontal force will be the sum of these two quantities: which, retaining only the first term in the expression when b is considered a small quantity, is

$$2Bb\ .\ \sin\ \theta\ .\ a^{-2}.$$

A similar expression with opposite sign gives the action on the blue pole (the needle being considered to be

small); and the algebraical difference or numerical sum of these gives the whole horizontal directive force

$$= 4Bb \,.\, a^{-3} \,.\, \sin\,\theta.$$

The resolved part of the action of the blue pole upon the red end of the needle, in the direction of the vertical at P, is

$$B \,.\, (a - b\,\cos\,\theta) \,.\, (a^2 + b^2 - 2\,ab\,\cos\,\theta)^{-} \;:$$

which, expanded to the first power of b, gives

$$B \,.\, a^{-3} \,.\, (a + 2b\,\cos\,\theta).$$

The action of the red pole upon the same red end of the needle is

$$-B \,.\, a^{-3} \,.\, (a - 2b\,\cos\,\theta).$$

The sum of these gives for the total vertical force downwards upon the red end,

$$4Bb \,.\, a^{-3} \,.\, \cos\,\theta.$$

As above, there is an opposite force, numerically additive, upon the blue end: and the whole vertical directive force is

$$8Bb \,.\, a^{-3} \,.\, \cos\,\theta.$$

Hence the tangent of dip at P

$$= \frac{\text{vertical force}}{\text{horizontal force}} = \frac{8Bb \,.\, a^{-3} \,.\, \cos\,\theta}{4Bb \,.\, a^{-3} \,.\, \sin\,\theta} = 2\,\text{cotan}\,\theta$$

$$= 2 \text{ tan magnetic latitude of } P.$$

And the total force at $P =$

$$\{(\text{hor. force})^2 + (\text{vert. force})^2\}^{\frac{1}{2}}$$

$$= 4Bb \,.\, a^{-3} \,.\, (\sin^2\,\theta + 4\,\cos^2\,\theta)^{\frac{1}{2}} \,.$$

At the magnetic equator, $\theta = 90^0$, and total force

$$= 4Bb \,.\, a^{-3}.$$

At the magnetic pole, $\theta = 0$, and total force

$$= 8Bb \,.\, a^{-3},$$

or double that at the equator.

These three results, for horizontal force, for dip, and for total force, are not materially disturbed if we conceive the magnet to be excentric, provided that magnetic latitude is always referred to its center.

It was soon found that this elegant theory, though well representing the broad facts of terrestrial magnetism, failed in accuracy when applied to many special cases. Such curves, for instance, as those of equal dip, Figures 20 and 21, could not possibly be explained by it. It was modified by supposing the axis of the magnet to be distant from the earth's center by one-seventh part of the earth's radius; but it could not then be sufficiently reconciled with observations.

46. *Attempt to explain Terrestrial Magnetism by the action of two magnets within the Earth.*

A celebrated Norwegian magnetical observer, Hansteen, remarking the tendency to the exhibition of two poles in the north and two poles in the south which we have indicated as appearing in some of the diagrams, Figures 20, 21, 28, 29, 35, 36, undertook the task of investigating the effects of two large magnets within the earth, both magnets being excentric, and inclined to the Earth's equator in different planes.

The investigations are contained in a work entitled *Magnetismus der Erde.* It will readily be conceived that this is a problem of great complexity. A great number of positions of the magnets were tried, but no one of them was quite satisfactory, though the results were superior to those derived from a single magnet.

As nothing has really resulted from this theory, it does not appear desirable to load the present Treatise with its laborious investigations. We may however remark that the known phænomena of observation amply justified the undertaking; and that, if it had not been made, we should often have felt that one possible opportunity of explaining Terrestrial Magnetism had been rejected.

47. *Gauss's more general explanation of Terrestrial Magnetism by supposing that the red and blue magnetisms are distributed irregularly through the earth.*

The investigation of this theory is given by Gauss in the *Resultate &c. des Magnetischen Vereins* for the year 1838; and a complete English translation of it is published in Taylor's Scientific Memoirs, volume ii. We shall not attempt here to explain all the generalities of this most elegant treatise. It will be sufficient to point out those parts which lead ultimately to the comparison of the results of theory with observation of the most extensive and most accurate kind.

It is supposed, as a law to which we are led by previous magnetic investigations, that the quantities

of red and of blue magnetism in the Earth are equal. And it is supposed that the attraction or repulsion is inversely as the square of the distance. The magnetism of every point of the Earth will be supposed, in the algebraical investigation, to be red: blue magnetism being included in the same investigation by conceiving its sign to be negative. As regards the experimental magnet or compass-needle, whose dimensions are exceedingly small in proportion to the distance of the magnetic parts of the Earth, it will be sufficient to consider the terrestrial action upon its red end only.

Let a, b, c be the coordinates of an attracting point: $\delta\mu$ the amount of magnetism there (its unit being that quantity of red magnetism which at distance 1 exercises on a similar mass the moving force represented by 1): and let x, y, z be the coordinates of the red end of the needle. The magnetic force on the end of the needle is $\frac{\delta\mu}{\rho^2}$, in the direction of the line joining the attracting and attracted points, where $\rho = \sqrt{\{(x-a)^2 + (y-b)^2 + (z-c)^2\}}$. Resolving this in the directions of x, y, z, the several forces are

$$\frac{\delta\mu \,.\, (x-a)}{\rho^3}, \quad \frac{\delta\mu \,.\, (y-b)}{\rho^3}, \quad \frac{\delta\mu \,.\, (z-c)}{\rho^3}.$$

It is easily seen that those forces are the same as

$$-\delta\mu \,.\, \frac{d\left(\frac{1}{\rho}\right)}{dx}, \quad -\delta\mu \,.\, \frac{d\left(\frac{1}{\rho}\right)}{dy}, \quad -\delta\mu \,.\, \frac{d\left(\frac{1}{\rho}\right)}{dz}.$$

A similar system of formulæ applies to the effects of

every magnetic particle. For summing the effects of the whole, let $V = -\int \delta\mu \cdot \frac{1}{\rho}$; then the total forces upon the particle x, y, z in the directions x, y, z, which we may call X, Y, Z, are respectively

$$\frac{dV}{dx}, \quad \frac{dV}{dy}, \quad \frac{dV}{dz}.$$

The symbol V will be recognized here as denoting the Potential of the forces acting on the particle x, y, z, affected with the negative sign: it is a physical quantity whose numerical value is independent of the directions of the ordinates x, y, z, provided they are rectangular.

Now instead of defining the place of the experimental needle by x, y, and z, it is convenient to define it by u the colatitude of the place or its angular distance from the terrestrial pole, λ the longitude of the place as measured from a fixed meridian towards the east, and r the distance of the place from the Earth's center. And it is convenient to estimate the magnetic forces in directions opposite to the directions of those coordinates as they are seen at the locality: namely as a force N towards the north, as a force W towards the west, and a force C in the vertical along the radius towards the Earth's center (the Earth being considered spherical). These three directions are at right angles to each other; and therefore they can be considered as the x, y, z of the last paragraph, and the expressions $\frac{dV}{dx}$ &c. can be employed, provided that we use

proper caution in interpreting these values in reference to our new polar coordinates. Now, considering the fictitious x as in the horizontal plane and towards the west, the value δx $\left(\text{by which } \frac{\delta V}{\delta x} \text{ and } \frac{dV}{dx} \text{ are formed}\right)$, is $-r \,.\, \sin u \,.\, \delta\lambda$; and therefore the westerly force $\frac{dV}{dx}$ will be $-\frac{1}{r \,.\, \sin u} \,.\, \frac{dV}{d\lambda}$. Considering the fictitious y as in the horizontal plane and towards the north, δy is $-r \,.\, \delta u$; and therefore the northerly force $\frac{dV}{dy}$ will be $-\frac{1}{r} \,.\, \frac{dV}{du}$. And considering the fictitious z as vertical at the place, δz is $-\delta r$; and therefore the vertical force $\frac{dV}{dz}$ is $-\frac{dV}{dr}$. Thus we have

$$N = -\frac{1}{r} \,.\, \frac{dV}{du},$$

$$W = -\frac{1}{r \,.\, \sin u} \,.\, \frac{dV}{d\lambda},$$

$$C = -\frac{dV}{dr}.$$

The algebraist may perhaps prefer a more rigorous investigation, of the following form.

Conceiving the place of observation on the globe as turned in some measure towards the spectator, the origin of longitude being to the extreme left hand or west; let x be measured from the Earth's center in the plane of equator towards the left; y in the plane of

equator towards the spectator: and z towards the north pole. Then

$$x = r \,.\, \sin u \,.\, \cos \lambda,$$

$$y = r \,.\, \sin u \,.\, \sin \lambda,$$

$$z = r \,.\, \cos u.$$

For changing our coordinates, we must put

$$\frac{dV}{dx} = \frac{dV}{du} \cdot \frac{du}{dx} + \frac{dV}{d\lambda} \cdot \frac{d\lambda}{dx} + \frac{dV}{dr} \cdot \frac{dr}{dx},$$

and similar equations for y and z: where u, λ, and r, are supposed to be explicitly expressed (as was V) in terms of x, y, z. Now $\tan^2 u = \frac{x^2 + y^2}{z^2}$; from which, after due reductions,

$$\frac{du}{dx} = \frac{1}{r} \cos u \,.\, \cos \lambda,$$

$$\frac{du}{dy} = \frac{1}{r} \cos u \,.\, \sin \lambda,$$

$$\frac{du}{dz} = -\frac{1}{r} \sin u.$$

And $\tan \lambda = \frac{y}{x}$; from which

$$\frac{d\lambda}{dx} = -\frac{1}{r} \cdot \frac{\sin \lambda}{\sin u},$$

$$\frac{d\lambda}{dy} = \frac{1}{r} \cdot \frac{\cos \lambda}{\sin u},$$

$$\frac{d\lambda}{dz} = 0.$$

And $r^2 = x^2 + y^2 + z^2$; whence

$$\frac{dr}{dx} = \sin u \,.\, \cos \lambda,$$

$$\frac{dr}{dy} = \sin u \,.\, \sin \lambda,$$

$$\frac{dr}{dz} = \cos u.$$

And thus

$$X = \frac{dV}{dx} = + \frac{dV}{du} \frac{1}{r} \cos u \,.\, \cos \lambda - \frac{dV}{d\lambda} \frac{1}{r} \,.\, \frac{\sin \lambda}{\sin u} + \frac{dV}{dr} \,.\, \sin u \cos \lambda,$$

$$Y = \frac{dV}{dy} = + \frac{dV}{du} \,.\, \frac{1}{r} \cos u \,.\, \sin \lambda + \frac{dV}{d\lambda} \,.\, \frac{1}{r} \,.\, \frac{\cos \lambda}{\sin u} + \frac{dV}{dr} \,.\, \sin u \,.\, \sin \lambda,$$

$$Z = \frac{dV}{dz} = - \frac{dV}{du} \,.\, \frac{1}{r} \sin u \quad + \frac{dV}{dr} \,.\, \cos u.$$

Now $W = X \sin \lambda - Y \cos \lambda$. Also the force in the direction of radius of the parallel passing through the point of observation $= X \cos \lambda + Y \sin \lambda$; from which, combined with the force Z,

$$N = Z \sin u - (X \cos \lambda + Y \sin \lambda) \cos u,$$

$$C = Z \cos u + (X \cos \lambda + Y \sin \lambda) \sin u.$$

Substituting the values of X, Y, Z,

$$N = - \frac{1}{r} \,.\, \frac{dV}{du},$$

$$W = - \frac{1}{r \sin u} \,.\, \frac{dV}{d\lambda},$$

$$C = - \frac{dV}{dr};$$

the same as the values found before.

Every thing now depends on the function V: and

this depends on $\frac{1}{\rho}$ or $\{(x-a)^2+(y-b)^2+(z-c)^2\}^{-\frac{1}{2}}$. For x, y, z, the ordinates of the experimental magnet, put their values (already used) $r \cdot \sin u \cdot \cos \lambda$, $r \sin u \cdot \sin \lambda$, $r \cos u$. And for a, b, c, the coordinates of a disturbing particle of magnetism, put similar coordinates, $a = r_0 \cdot \sin u_0 \cdot \cos \lambda_0$, $b = r_0 \cdot \sin u_0 \cdot \sin \lambda_0$, $c = r_0 \cdot \cos u_0$. (If the experimental magnet be on the earth's surface, and the disturbing magnetism be within the earth, r_0 is always less than r.) The value of $\frac{1}{\rho}$ now becomes $[r^2 - 2r\, r_0 \{\sin u \cdot \sin u_0 \cdot \cos(\lambda - \lambda_0) + \cos u \cdot \cos u_0\} + r_0^2]^{-\frac{1}{2}}$; which can be expanded in a converging series

$$\frac{1}{r}\left\{T_0 + T_1\left(\frac{r_0}{r}\right) + T_2\left(\frac{r_0}{r}\right)^2 + T_3\left(\frac{r_0}{r}\right)^3 + \&c.\right\},$$

where $T_0 = 1$, and T_1, T_2 &c., are functions only of u, u_0, and $\lambda - \lambda_0$. Put R for the earth's radius (the symbol r being still reserved for the radius at the place of observation, in order to preserve the generality which admits of differentiation with respect to r). Then V or $-\int \delta\mu \cdot \frac{1}{\rho}$ may be put in the form

$$\frac{R^2 P_0}{r} + \frac{R^3 P_1}{r^2} + \frac{R^4 P_2}{r^3} + \frac{R^5 P_3}{r^4} + \&c.,$$

where $R^2 P_0 = -\int T_0 \cdot \delta\mu$, $R^3 P_1 = -\int T_1 \cdot r_0\, \delta\mu$, $R^4 P_2 = -\int T_2\, r_0^2\, \delta\mu$, &c. The general term will be

$$\frac{R^{n+2} P_n}{r^{n+1}},$$

where $R^{n+2} P_n = -\int T_n \cdot r_0^n \cdot \delta\mu$.

Now forming the values of N, W, C, and remarking that (as the integral with respect to $\delta\mu$ applies only to elements entirely independent of u, λ, and r,) the differentiations with respect to u and λ can be performed under the integral sign, and the differentiation with respect to r will be entirely external to the integral sign; the general term of N or $-\frac{1}{r}.\frac{dV}{du}$ will be

$$-\frac{R^{n+2}}{r^{n+2}}\cdot\frac{dP_n}{du};$$

that of W or $-\frac{1}{r\sin u}.\frac{dV}{d\lambda}$ will be

$$-\frac{1}{\sin u}.\frac{R^{n+2}}{r^{n+2}}\cdot\frac{dP_n}{d\lambda};$$

that of C or $-\frac{dV}{dr}$ will be

$$+(n+1).\frac{R^{n+2}}{r^{n+2}}.P_n.$$

Also it is to be remarked that T_0 is 1, and therefore $\int T_0\,\delta\mu$ is 0 (because the total amounts of red and of blue magnetism are supposed to be equal), and therefore P_1 is 0. And, if our needle be on the earth's surface, $r=R$. Thus we obtain

$$N=-\frac{dP_1}{du}-\frac{dP_2}{du}-\&c.-\frac{dP_n}{du}-\&c.$$

$$W=-\frac{1}{\sin u}\left\{\frac{dP_1}{d\lambda}+\frac{dP_2}{d\lambda}+\&c.+\frac{dP_n}{d\lambda}+\&c.\right\}.$$

$$C=+2P_1+3P_2+\&c.+(n+1)P_n+\&c.$$

where $r^{n+2}.P_n=-\int T_n.r_0^n.\delta\mu$,

and T_n is the coefficient (in terms of $u, u_0, \lambda, \lambda_0$) of $\left(\frac{r_0}{r}\right)^n$ in the development of $\frac{1}{\rho}$.

48. *Incidental introduction of Laplace's Coefficients (not further used in this Treatise).*

If we differentiate twice the expression

$$\frac{1}{\rho} = \{(x-a)^2 + (y-b)^2 + (z-c)^2\}^{-\frac{1}{2}}$$

with respect to x, also with respect to y, and with respect to z, we find

$$\frac{d^2\left(\frac{1}{\rho}\right)}{dx^2} + \frac{d^2\left(\frac{1}{\rho}\right)}{dy^2} + \frac{d^2\left(\frac{1}{\rho}\right)}{dz^2} = 0.$$

And since $V = -\int \delta\mu \frac{1}{\rho}$, and since the application and limits of this integration do not depend on x, y, z, it follows that

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0.$$

Now, by the same principle which we have used in the last Article for V, and which we shall here use successively for $\frac{dV}{dx}$, $\frac{dV}{dy}$, and $\frac{dV}{dz}$,

$$\frac{d^2V}{dx^2} = \frac{d\left(\frac{dV}{dx}\right)}{du} \cdot \frac{du}{dx} + \frac{d\left(\frac{dV}{dx}\right)}{d\lambda} \cdot \frac{d\lambda}{dx} + \frac{d\left(\frac{dV}{dx}\right)}{dr} \cdot \frac{dr}{dx};$$

$$\frac{d^2V}{dy^2} = \frac{d\left(\frac{dV}{dy}\right)}{du}\cdot\frac{du}{dy} + \frac{d\left(\frac{dV}{dy}\right)}{d\lambda}\cdot\frac{d\lambda}{dy} + \frac{d\left(\frac{dV}{dy}\right)}{dr}\cdot\frac{dr}{dy};$$

$$\frac{d^2V}{dz^2} = \frac{d\left(\frac{dV}{dz}\right)}{du}\cdot\frac{du}{dz} + \frac{d\left(\frac{dV}{dz}\right)}{d\lambda}\cdot\frac{d\lambda}{dz} + \frac{d\left(\frac{dV}{dz}\right)}{dr}\cdot\frac{dr}{dz}.$$

The expressions for all the quantities on the right hand are to be found in the last Article: those for $\frac{dV}{dx}$ &c. requiring complete differentiation, with special notice that $\frac{dV}{du}$ &c. are themselves subject to differentiation, as well as the more explicit quantities cos u, &c. Performing all operations, this result is obtained:—

$$\frac{d^2(rV)}{dr^2} + \frac{1}{r}\cdot\frac{\cos u}{\sin u}\cdot\frac{dV}{du} + \frac{1}{r}\cdot\frac{d^2V}{du^2} + \frac{1}{r\sin^2 u}\cdot\frac{d^2V}{d\lambda^2} = 0.$$

Applying this to the general term $\frac{R^{n+2}P_n}{r^{n+1}}$, where P_n is independent of r,

$$\frac{n(n+1).P_n}{r^{n+2}} + \frac{1}{r^{n+2}}\cdot\frac{\cos u}{\sin u}\cdot\frac{dP_n}{du} + \frac{1}{r^{n+2}}\cdot\frac{d^2P_n}{du^2} + \frac{1}{r^{n+2}\sin^2 u}\cdot\frac{d^2P_n}{d\lambda^2} = 0;$$

or $$n(n+1).P_n + \frac{\cos u}{\sin u}\cdot\frac{dP_n}{du} + \frac{d^2P_n}{du^2} + \frac{1}{\sin^2 u}\cdot\frac{d^2P_n}{d\lambda^2} = 0;$$

from which it is possible to find a general expression for P_n. The terms thus found are Laplace's Coefficients. In the physical investigation now before us, we shall not have occasion to use the general term.

49. *Continuation of Gauss's investigation: application in a numerical form.*

Put p for $\sin u \,.\, \sin u_0 \,.\, \cos(\lambda - \lambda_0) + \cos u \,.\, \cos u_0$, and expand the fraction for $\frac{1}{\rho}$. In the paucity of well-determined elements, and in the complexity of expressions, Gauss thought it sufficient to develope this to the 4th power of r_0. This gives for $\frac{1}{\rho}$,

$$\frac{1}{r} + p\frac{r_0}{r^2} + \left(-\frac{1}{2} + \frac{3}{2}p^2\right)\frac{r_0^2}{r^3} + \left(-\frac{3}{2}p + \frac{5}{2}p^3\right)\frac{r_0^3}{r^4} + \left(\frac{3}{8} - \frac{15}{4}p^2 + \frac{35}{8}p^4\right)\frac{r_0^4}{r^5}.$$

Now $\frac{pr_0}{r^2} =$

$$\frac{\cos u \,.\, \cos\lambda}{r^2} \,.\, (r_0 \,.\, \cos u_0 \,.\, \cos\lambda_0) + \frac{\cos u \,.\, \sin\lambda}{r^2}(r^0 \,.\, \cos u_0 \,.\, \sin\lambda_0) + \frac{\sin u}{r^2} \,.\, (r_0 \,.\, \sin u_0).$$

The corresponding term of V will be

$$-\frac{\cos u \,.\, \cos\lambda}{r^2}\int \delta\mu \,.\, r_0 \,.\, \cos u_0 \,.\, \cos\lambda_0$$

$$-\frac{\cos u \,.\, \sin\lambda}{r^2}\int \delta\mu \,.\, r_0 \,.\, \cos u_0 \,.\, \sin\lambda_0 - \frac{\sin u}{r^2}\int \delta\mu \,.\, r_0 \,.\, \sin u_0.$$

Each of these integrals is an unknown constant. Calling them i_1, i_2, i_3, the term of V will be

$$-\frac{\cos u \,.\, \cos\lambda}{r^2} \,.\, i_1 - \frac{\cos u \,.\, \sin\lambda}{r^2} \,.\, i_2 - \frac{\sin u}{r^2} \,.\, i_3,$$

where for any special locality on the earth, u, λ, and r, must have the proper numerical values, but i_1, i_2, i_3 must for the present be left in a symbolical form. The expansions of p^2, p^3, &c., will introduce other integrals or unknown constants i_4, i_5, i_6, &c. multiplied by other functions of u, λ, and r. And thus the forces N, W, C, can be exhibited for every locality, in expressions which involve these unknown constants: then the westerly declination, whose tangent $= \frac{W}{N}$, can be so expressed: the total horizontal magnetic force $= \sqrt{(N^2 + W^2)}$ can be so expressed: and the angle of dip, whose tangent $= \frac{C}{\sqrt{(N^2 + W^2)}}$, can be so expressed.

The number of integrals or undetermined constants thus introduced is large. Limiting the order (as above mentioned) to P_4 or to the fourth power of p, 24 constants are required. In order to obtain these numerically, 24 observations of some kind are necessary. Any determinations of magnetic elements will suffice: for instance, determinations of western declination, horizontal force, and dip, at each of eight stations. Gauss, referring generally to Sabine's map of Total Intensity in the *Seventh Report of the British Association*, and to Barlow's map of Declination, *Phil. Trans.* 1833, and to Horner's map, *Physikalisches Wörterbuch*, Band VI, but without giving numerical details of his process, has obtained the following value for $\frac{V}{R}$. It is to be remarked that the numbers have all been adapted to give horizontal force

at London = 1732 (it having been customary in former times to call that force 1·732). The first or constant term, which does not appear in our formula for V, probably arises from the conversion of powers of cosines &c. into cosines of multiple arcs. The letter e stands for cos u and f for sin u.

$$\frac{V}{R} = -1{\cdot}977 + 937{\cdot}103\,e + 71{\cdot}245\,e^2 - 18{\cdot}868\,e^3 - 108{\cdot}855\,e^4$$
$$+ (64{\cdot}437 - 79{\cdot}518\,e + 122{\cdot}936\,e^2 + 152{\cdot}589\,e^3)\,f\cos\lambda$$
$$+ (-188{\cdot}303 - 33{\cdot}507\,e + 47{\cdot}794\,e^2 + 64{\cdot}112\,e^3)\,f\sin\lambda$$
$$+ (7{\cdot}035 - 73{\cdot}193\,e - 45{\cdot}791\,e^2)\,f^2\cos 2\lambda$$
$$+ (-45{\cdot}092 - 22{\cdot}766\,e - 42{\cdot}573\,e^2)\,f^2\sin 2\lambda$$
$$+ (1{\cdot}396 + 19{\cdot}774\,e)\,f^3\cos 3\lambda + (-18{\cdot}750 - 0{\cdot}178\,e)\,f^3\sin 3\lambda$$
$$+ 4{\cdot}127\,f^4\cos 4\lambda + 3{\cdot}175\,f^4\sin 4\lambda.$$

From these, by the formulæ in Article 47, are formed numerical values of N, W, and C, for numerous latitudes and longitudes: and from them are derived numerical values of Declination, Horizontal Force, and Dip. By means of the values of Declination, curves of Places of Equal Declination were laid down by Gauss upon a map, from which the writer of this Treatise has formed the Magnetic Meridians in Figures 20 and 21. The Magnetic Meridians may also be traced by the following process. Conceive fictitious ordinates x, y, z, as in Article 47, where x and y are on the tangent-plane of any point on the earth's surface, and x is in the direction in which V does not alter, that is, in the direction of a curve of Equal Values of V. The general

expression for force in the direction x is $\frac{dV}{dx}$. But in this instance, V does not alter with alteration of x; therefore $\frac{dV}{dx}$ is 0, and there is no force in the direction of x. Consequently the whole horizontal force is in the direction of y, or perpendicular to the curve of Equal Values of V. Now Gauss has prepared curves of Equal Values of V (not copied in this Treatise), and therefore it is only necessary to draw trajectory-curves cutting all the Equal-V-curves at right angles, and the lines so drawn will be the lines of direction of total horizontal force, or the Magnetic Meridians.

The curves of Equal Horizontal Force in Figures 28 and 29, those of Equal Total Intensity in Figures 35 and 36, and those of Equal Dip in Figures 20 and 21, are copied immediately from Gauss.

The elements from which these curves have been formed having been deduced from 24 magnetic measures made at different places, those measures are necessarily exhibited correctly in the curves. And now the question arises, whether all other measures made since that time are exhibited accurately by the curves. And the answer is, that they are exhibited so accurately as to leave no doubt on the fundamental correctness of the theory, and yet with small discordances which render it desirable that the formulæ should be extended and compared with a greater number of measures for numerical determination of the constants in $\frac{V}{R}$. In the

Astronomische Nachrichten, Nos. 1792 and 1793, H. Peterson has given the results derived from 610 measures: but the symbolical development is still limited (as above) to the 4th order. The results are not yet exhibited in a form easily understood by the eye.

We cannot terminate this Section of our work without earnestly inviting the attention of our readers to the whole of Gauss's investigation: one of the most beautiful and the most important that has appeared for many years in Physical Mathematics.

Section VII.

Disturbing force produced on a small compass-needle by a large magnet, in various positions; and composition of this disturbing force with terrestrial horizontal force.

50. *The disturbing magnet is horizontal: its center is broadside-on to the center of the compass: to find its effect at different distances and elevations.*

In this and subsequent investigations of this Section, we shall consider the dimensions of the compass-needle to be so small, in comparison with other measures, that we may use the lengths of lines measured to the center of the compass-needle instead of those measured to its poles: and we shall investigate the action upon the red end only of the compass-needle, inasmuch as the action on the blue end will be sensibly equal but in the opposite direction, and the impressed moment of rotation will be merely doubled.

In Figure 39, the attraction of the blue pole of A and the repulsion of the red pole produce in the direc-

tion of c the respective forces

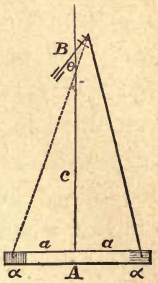
$$+\alpha . c . (a^2+c^2)^{-\frac{3}{2}}, \quad -\alpha . c . (a^2+c^2)^{-\frac{3}{2}},$$

which destroy each other: but they produce in the direction towards the right the forces

$$+\alpha . a(a^2+c^2)^{-\frac{3}{2}}, \quad +\alpha . a(a^2+c^2)^{-\frac{3}{2}},$$

which are to be added together, producing $2\alpha . a(a^2+c^2)^{-\frac{3}{2}}$

Fig. 39.



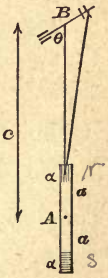
acting towards the right. This force, it is to be observed, is parallel to the length of A or perpendicular to the length of c, whatever be the plane containing the center of the compass and the axis of A. The plane αaB may be horizontal, or vertical, or inclined, but the expression found above applies to every one of these cases.

51. *The disturbing magnet is end-on to the compass: first, in the horizontal plane: secondly, in an inclined plane, the axis of the magnet still directed to the compass.*

In Figure 40, the attraction of the blue end of the magnet on the red end of the compass is represented by $\frac{\alpha}{(c-a)^2}$, and the repulsion of the red end by $\frac{\alpha}{(c+a)^2}$; the difference is

$$4\alpha . \frac{ca}{(c^2-a^2)^2} .$$

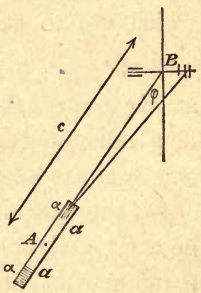
Fig. 40.



This, in every case of the magnetic end directed towards the compass, represents truly the entire action, tending (with the poles as supposed in the figure) to draw the red end of the compass towards A. And, when A is contained in a horizontal plane passing through B, the expression gives the force which tends to disturb the red end of the compass in the horizontal plane.

But when the direction of A is inclined to the horizon, take the vertical plane passing through A as in Figure 41; the force which A exerts on the red end of the compass is that just found, but it acts in the direction BA: and the horizontal part of it will be obtained by multiplying by $\sin\phi$, or will be

Fig. 41.



$$4\alpha\frac{ca}{(c^2-a^2)^2}\cdot\sin\phi.$$

52. *The disturbing magnet is horizontal; it is directed end-on to the vertical axis of the compass, and is not necessarily at the same elevation as the compass.*

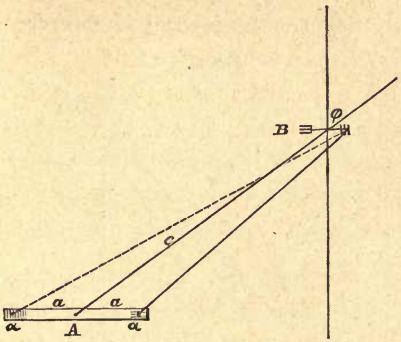
This state of things is represented in Figure 42. The attraction of the blue end is $\dfrac{\alpha}{c^2+a^2-2ca\sin\phi}$: and the horizontal part of this is

$$a(c\sin\phi-a)\,(c^2+a^2-2ca\sin\phi)^{-\frac{3}{2}}.$$

The horizontal repulsion of the red end is

$$a(c\sin\phi+a)\,(c^2+a^2+2ca\sin\phi)^{-\frac{3}{2}}.$$

Fig. 42.



The effective attraction in the horizontal plane is the excess of the former over the latter.

If the magnet be at a considerable distance, or if a be much smaller than c, so that it will suffice to include the first power of a, an approximate value will be

$$\frac{\alpha}{c^5}(c\sin\phi - a)(c^2 + 3ca\sin\phi) - \frac{\alpha}{c^5}(c\sin\phi + a)(c^2 - 3ca\sin\phi);$$

or, nearly,

$$\frac{\alpha}{c^5}(c^3\sin\phi + 3c^2a\sin^2\phi - c^2a) - \frac{\alpha}{c^5}(c^3\sin\phi - 3c^2a\sin^2\phi + c^2a);$$

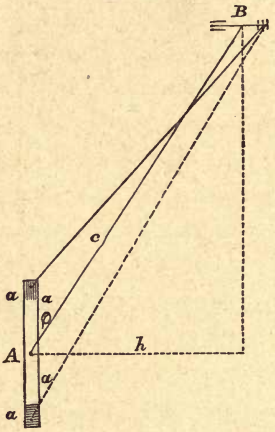
or
$$\frac{2a\alpha}{c^3}(3\sin^2\phi - 1).$$

Consequently, when $\sin\phi = \sqrt{\frac{1}{3}}$, or $\phi = 35^\circ\ 16'$, there is no horizontal action: when ϕ is less than this angle, the action is of the opposite character.

53. *The disturbing magnet is vertical.*

In Figure 43, the horizontal part of the attraction

Fig. 43.



of the blue pole is $\alpha \,.\, c \,.\, \sin\phi \,.\, (c^2 + a^2 - 2ca\cos\phi)^{-\frac{3}{2}}$, and that of the repulsion of the red pole is

$$\alpha \,.\, c \sin\phi \,(c^2 + a^2 + 2ca\cos\phi)^{-\frac{3}{2}}.$$

If these be expanded to the first power of a, the resultant attraction is found to be

$$\frac{6a\alpha}{c^3} \,.\, \sin\phi \,.\, \cos\phi.$$

With a given value of c, it is therefore greatest when $\phi = 45°$.

With a given horizontal distance h, which makes $\frac{1}{c^3} = \frac{\sin^3\phi}{h^3}$, the force is

$$\frac{6a\alpha}{h^3} \,.\, \sin^4\phi \,.\, \cos\phi\,;$$

this is greatest when $\tan \phi = 2$, which gives the depression of the center of the magnet below the compass $= \frac{h}{2}$.

54. *The disturbing magnet is in the horizontal plane which passes through the compass, but is inclined at any angle to the line joining the centers of the magnets.*

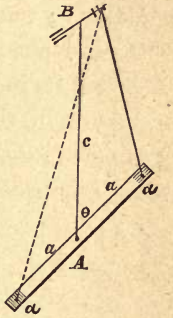
In Figure 44, it will readily be seen that the resolved part of the force in the direction of c is

$$\alpha . (c - a\cos\theta)\,(c^2 - 2ca\cos\theta + a^2)^{-\frac{3}{2}}$$
$$-\alpha . (c + a\cos\theta)\,(c^2 + 2ca\cos\theta + a^2)^{-\frac{3}{2}}:$$

and the resolved part perpendicular to c towards the right is

$$\alpha . a\sin\theta(c^2 - 2ca\cos\theta + a^2)^{-\frac{3}{2}}$$
$$+\alpha . a\sin\theta(c^2 + 2ca\cos\theta + a^2)^{-\frac{3}{2}}.$$

Fig. 44.



If we expand these to the third power of $\frac{a}{c}$, we find,

Force in the direction of c

$$= \frac{4a\alpha}{c^3}\cos\theta\left\{1 + \frac{a^2}{c^2}(5\cos^2\theta - 3)\right\};$$

Force transversal to c

$$= \frac{2a\alpha}{c^3}\sin\theta\left\{1 + \frac{a^2}{c^2}\left(\frac{15}{2}\cos^2\theta - \frac{3}{2}\right)\right\}.$$

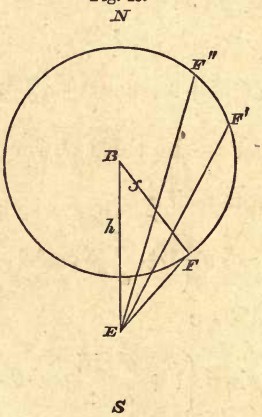
The square root of the sum of the squares of these will give the whole force on the red end of the compass-

needle, and the quotient of the second by the first will give the tangent of the inclination of the whole force to *c*.

55. *Composition of the disturbing force in the horizontal plane with the terrestrial horizontal force.*

In the case of Figure 39, the horizontal force produced by *A* is in the direction at right angles to *c*: in Figures 40, 41, 42, 43, it is in the vertical plane which contains *c*; and in Figure 44, it makes a definite angle with *c*, depending only on the magnitude and distance of *A* and its inclination to *c*. If the disturbing magnet *A* rotate in the horizontal plane (as for instance when it is part of a ship revolving in azimuth, the compass in these figures being the ship's compass), in Figure 45,

Fig. 45.



let *BF* with length *f* represent the force which, as found above, is produced by the magnet *A* acting on the red

end of B; and let EB with length h, as measured from E to B, represent the earth's horizontal magnetic force acting on the same red end of B. Then EF will represent in magnitude and direction the total composite horizontal force acting on the red end of the compass-needle. As the ship rotates in azimuth, the line f will assume the different positions BF', BF'', &c., the points F', F'', &c. all lying in a circle of which B is the center: and in these different positions, the total horizontal force acting on the needle will be represented in magnitude and direction by EF', EF'', &c. It is seen here that when the ship is in such a position that the deviation of the compass (which is the same as the angle BEF) $= 0$, the force is either the greatest possible $= h+f$, or the least possible $= h-f$. It is also seen that in the entire revolution of the ship, the compass-needle deviates during half of the revolution to the right and during the other half to the left. The positions of the ship at maximum deviation to the right and maximum deviation to the left are not exactly half-way between the positions of no deviation.

If f be greater than h, the circle will include the point B: and, as the ship revolves uniformly, the compass-needle will turn entirely round, but not with uniform angular velocity.

SECTION VIII.

On Transient Induced Magnetism in Soft Iron.

56. *Definition of Soft Iron, and criterion of the magnetic difference between Soft Iron and Magnetized Steel.*

Under the term Soft Iron may be understood, either Malleable Iron which has not been hammered or subjected to any violence when cold, or Cast Iron. (We shall in the next Section discuss the properties of Malleable Iron when subjected in the cold state to violence.) And the best practical criterion by which a bar of Soft Iron is distinguished from a Steel Magnet is this. We have found in Articles 16, 27, and other places, that if, in the horizontal plane, a steel magnet is applied end-on towards the center of a suspended horizontal magnet, it tends to produce a deviation in the position of the suspended magnet. Now if a bar of soft iron be substituted for the steel magnet, the suspended magnet will not be disturbed at all. In some positions, if the suspended magnet be constrained by external

force to take a position other than north and south (as for instance, if suspended by two threads as in the apparatus for measure of small changes of horizontal force, Article 85), the presentation to it of a bar of soft iron end-on to the center will slightly disturb it: but to a degree very much less than that of which we shall speak in the next article.

57. *Experiments on the induction of magnetism in Soft Iron by the action of a Steel Magnet.*

In Figures 46 and 48, suppose that A is a steel magnet in a vertical position (it matters little whether the red end is upwards or downwards: in the diagram it is supposed that the red end is upwards). In Figure 46 suppose that C is a small bar of soft iron (as a small nail) lying on a table so far below A that the action of A will not sensibly disturb C. Suppose that B in Figure 47 is a bar of soft iron (as a larger nail) which, alone, would not disturb C. Now let the bar B be placed under A as in Figure 48 (in which case the magnet A if sufficiently powerful will support B, the reason of which we shall hereafter explain), and B will immediately lift the small bar C. If the bar B be held in the left hand, and A in the right, then, upon detaching A from B, C will immediately drop off. On the other hand, if the connexion of A, B, and C, be maintained, C will support a piece of iron wire D, as in Figure 48. And this series may sometimes be continued through several steps.

Fig. 46.

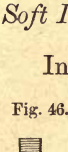


Fig. 47.

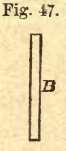
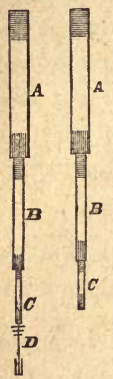
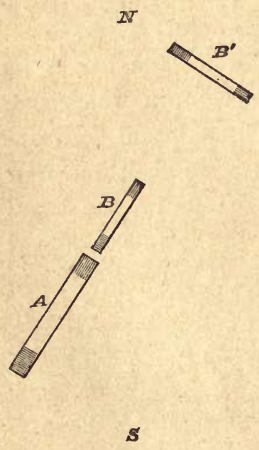


Fig. 48.



It is evident here that B is converted into a magnet as long as it is under the influence of A, and no longer. And this is the characteristic of Transient Induced Magnetism. If the quality of the magnetism of the lower end of B be examined by the disturbance which it produces in a compass-needle, it is found to be the same as that of the lower end of A (blue magnetism, in the diagram). This leads to the presumption, in analogy with other phænomena of magnetism, that the magnetism of the upper end of B is of the kind opposite to that of the lower end of A: a presumption which we shall find to be supported in the case which we can examine more perfectly, that of transient induction produced by the earth's action.

Fig. 49.



The same conclusions will be arrived at by examination of the deviation produced in a suspended magnet or compass-needle; as in Figure 49. If the magnet A has produced deviation of B' to the position shewn in the diagram, and the bar of soft iron B be inserted (under circumstances where, if alone, its effect on B' would be imperceptible), it greatly increases the deviation of B'.

The effect is considerable if B does not touch A, but much larger if B touches A. It is certain here that the nature of the magnetism in the advanced end of B is the same as that in the advanced end of A.

Or, if the magnet A be held vertically above the center of a small compass (in which state it will not disturb the compass); and if the upper end of B touch the magnet, and its lower end be carried conically round the compass: it will disturb it in a manner which shews that the lower end of B has the same kind of magnetism as the lower end of A.

58. *Explanation of the attraction of soft iron by either pole of a steel magnet, as an effect of induction.*

We are now in a position to explain the ordinary phænomenon, (perhaps the best known of all magnetic phænomena), of attraction of soft iron by either pole of a magnet. In Figure 48, B is, for the time, a magnet as well as A; and the two poles (that of A and that of B) which are in contact, have, one blue magnetism, the other red. Therefore there is attraction. It is seen that it is indifferent which pole of A is presented to B: a blue pole of A produces an adjacent red pole in B, or a red pole of A produces an adjacent blue pole in B: and in both cases there is attraction.

We see also that the phænomenon is entirely in accordance with that of the magnetization of steel by double-touch, Article 8. It appeared there that the blue magnetism of one end of the dominant magnet

dragged the red magnetism of the affected magnet to one end and there left it fixed: here it seems that it draws the red magnetism of the iron bar (or a portion of it) to one end, but cannot leave it fixed there: that in the instance of iron, as distinguished from steel, the separate kinds of magnetism take the earliest opportunity of returning to their original seats and producing neutral magnetism in every part.

Fig. 50.

We also see the reason why a horse-shoe magnet so energetically attracts a piece of iron touching both its poles, as in Figure 50. Each pole of the horse-shoe converts the corresponding part of the iron into a pole of opposite quality, and the existence of each impressed pole at one end of the iron seems to have a tendency to intensify the opposite pole at the other end, and thus the iron is in the state of a powerful magnet attracted by another powerful magnet, and the attraction (proportional to the product of the powers) is very energetic.

59. *Rapid diminution, with increase of distance, of the attraction between a magnet and soft iron.*

The magnetic power of the permanent magnetism in one pole of the magnet varies, as has been demonstrated, inversely as the square of the distance of the magnetic body on which it acts. It appears reasonable to suppose that its influence in inducing magnetism

follows the same law, and therefore that the energy of the induced magnetism is inversely as the square of the distance. Consequently, the attraction between the two magnetisms (the permanent magnetism of the magnet and the induced magnetism of the iron), which is as the product of these magnetisms directly and as the square of the distance inversely, will be inversely as the fourth power of the distance. With increase of distance therefore the attraction diminishes very rapidly.

When the distance is so far increased that the effect of the farther pole of the magnet, though diminished, is less diminished than that of the nearer pole, and becomes comparatively sensible, it tends still more to diminish the attraction. And, on the whole, the attraction diminishes with extreme rapidity, and is sensible only at very small distances.

60. *Induction of magnetism in soft iron, produced by terrestrial magnetism.*

Take a bar of soft iron, which for convenience of language we will suppose to have one end painted white and the other end black: hold it vertical, with the black end downwards. Upon applying any of the ordinary tests, it will instantly be found that the bar in this position is a genuine magnet, and that its black end is charged with red magnetism and its white end with blue magnetism. The easiest proof will be, holding it parallel to itself, to carry it round a small compass: if the black or lower end is at the level of the compass, it

attracts the blue end of the compass-needle: if the white or upper end is at the level of the compass, it attracts the red end of the needle: if the middle of its length is at the level of the compass, it produces no sensible disturbance.

Yet this magnetism does not imply any permanent modification in the state of the iron bar. For, invert the bar, so that the white end is downwards, and apply it in the same way to the experimental compass. Now, the white end of the bar attracts the blue end of the needle (instead of attracting the red end as it did before) and the black end of the bar attracts the red end of the needle (instead of attracting the blue end as it did before). The iron bar is for the time a magnet, but its poles are in the direction opposite, as regards the structure of the iron, to that in which they were before.

But they are in the same direction as regards *up* and *down*. The upper end (whether white or black) is always a blue pole, and the lower end, (whether black or white), is always a red pole.

These experiments are described as they are seen in the northern magnetic latitudes of the earth. In the southern magnetic latitudes, the lower end of the bar has blue magnetism. At the magnetic equator, the experiment fails in this form; but a slight variation in the form of the experiments, applicable in every place, exhibits the induced magnetism in the greatest possible intensity; the variation is merely the following:

Instead of holding the bar in the vertical position, hold it in the direction of the local dip. Then it will

be found that the quality of the magnetism of the bar is always the same as that of the dipping-needle.

Now vary the experiment by holding the bar so that its length is contained in the plane to which the dip-direction is normal. Its disturbing power ceases entirely: it has no sensible charge of magnetism.

All these phænomena are exactly similar to those described in Article 57, conceiving the earth's action to be similar to that of a steel magnet: and the explanation is the same as that in Article 58, that the attraction of the earth's red magnetism draws towards itself the blue magnetism which is in the particles of the iron: and similarly for the attraction of the blue on the red.

61. *Effect of the terrestrially-induced magnetism in a mass of soft iron which is carried round a compass, at the same level as the compass, and with the same part of the mass always directed to the compass-center.*

This case is one which theoretically deserves attention, and which in practical application is very important, inasmuch as in iron-built and other ships it represents the state of things where, partly from the iron of the ship-construction and partly from the iron introduced for corrective purposes, there is much iron admitting of induction from terrestrial magnetism, at nearly the same elevation as the compass, and revolving round it as the ship swings round, always presenting the same part to the compass.

Upon carrying the mass of iron round the compass in the manner described, the phænomena are these: When the central point of the mass (if symmetrically shaped), or a certain central point (in general) is on the N. or S. or E. or W. side of the compass-center, it produces no disturbance in the compass-needle. When the direction of that central point is between N. and E., it turns the N. end of the needle to the E.: when between E. and S., it turns it to the W.: when the central point of the mass is between S. and W., it turns the N. end of the needle to the E.: when between W. and N., it turns the N. end to the W. On comparing these with the deviations produced by a magnet which is carried round the compass in the same manner, as described in Article 55, it is seen that there is a striking difference; in the case of a complete revolution of the magnet, the needle is made to deviate once to the right and once to the left; but in the case of a complete revolution of the soft iron, the needle deviates twice to the right and twice to the left. If the azimuth of the disturbing mass, as viewed from the center of the compass, and measured from N. towards E. be called θ, the amount of deviation produced in the needle from N. towards E. is exactly or approximately proportional to $\sin 2\theta$: vanishing when θ is 0°, 90°, 180°, 270°, and becoming negative when θ is $> 90^\circ < 180^\circ$, or $> 270^\circ < 360^\circ$. The law of disturbance may be represented (for memory only) by this rule: the mass attracts that pole of the needle which is nearest to it.

62. *Effect of the combination of two masses of iron, in opposite azimuths: and of two masses of iron, in azimuths differing* 90°.

One curious consequence of this law, easily verified in experiment, is, that if a mass similar to the first mass be placed on the opposite side of the compass, carried by the same frame so that in revolution it is always opposite to the first mass, it doubles the disturbance; but if it is placed at 90° either to the right or to the left of the original mass, always retaining that relative position, it neutralizes the disturbance. For, the original disturbance being $a \sin 2\theta$, that of an opposite mass will be $a \sin 2(\theta + 180^\circ)$

$$= a \sin 2\theta,$$

the addition of which doubles the first: but the disturbance produced by a mass 90° to right or left will be $a \sin 2(\theta \pm 90^\circ)$

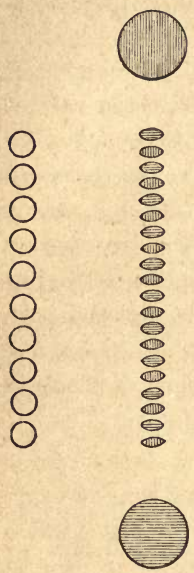
$$= -a \sin 2\theta,$$

the addition of which neutralizes the first.

63. *Simplest form of theory for explanation of the phœnomena of induction.*

In Figure 51 conceive the first line of circles to represent particles of a mass of iron, or at least so many of the particles as contain united portions of red magnetism and blue magnetism, in a line extending through a mass of iron. And conceive

Fig. 51.



the second line to represent the state of their magnetisms as affected by the induction of the great masses of blue and red magnetism external to them. (The effect of one of these masses alone is precisely similar in kind to that of the two masses.) Then, in analogy with everything that we have seen of magnetization of steel magnets and of iron bars, we may conceive the blue magnetism of each circle to be drawn towards the external red mass, and the red magnetism of each circle to be drawn towards the external blue mass, as shewn in the figure. The effect of this will be that, through all the intermediate parts of the series, the blue and red alternate in such a way that we cannot perceive any clear tendency in them to produce magnetic effect on an external body: but there is certainly a red pole at one end and certainly a blue pole at the other. When we conceive a system of parallel lines of the same kind passing through a mass of iron, we find that the whole exterior surface which is turned towards the great red mass is clothed with blue magnetism, and that the whole which is turned towards the great blue mass is clothed with red magnetism: and the mass resembles to some extent a steel magnet.

64. *The inductive energy may be resolved in different directions, in the same manner as statical forces.*

Fig. 52.

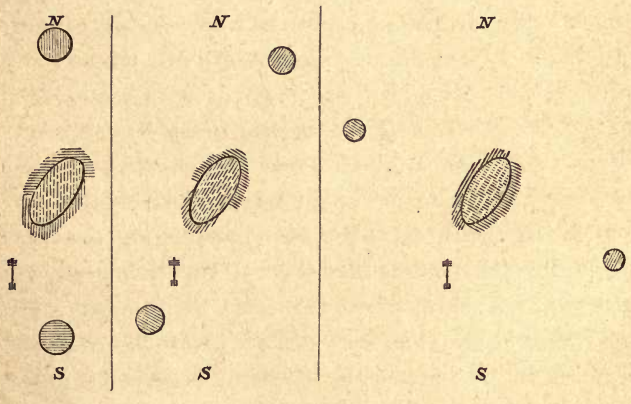
In Figure 52, let a and b represent the separated masses of magnetism of equal intensity produced by one of the small circles in Figure 51. It seems reasonable to suppose that the extent of their separation will be proportional to the external magnetic energy. Take the positions $b'a'$ (coincident in space) for two masses of opposite magnetisms, each equal to a or b. These two masses, while coexisting, neutralize each other. But we may conceive b' associated with a and a' associated with b; and we may consider the pair ab' as the effect of one inducing magnetism in the direction ab', and the pair $a'b$ as the effect of another inducing magnetism in the direction $a'b$; and the magnitudes of the two inducing magnetisms must (by the general assumption mentioned above) be considered proportional to the lengths ab', $a'b$. It is seen here that we have in fact resolved the primary inducing energy into two, according to the laws of resolution of statical forces: and if, in any proposed problem, it can be shewn that one of these is inefficient, we may confine our attention to the other: or if the effects of the resolved inductions can be computed more easily than that of the

original induction we may use them instead of the original induction.

65. *A mass of iron, symmetrical with respect to the plane directed to the axis of a compass and with respect to the horizontal plane, and with its center at the same height as the compass, is subject to terrestrial induction: theoretical investigation of its deviating energy on the compass: it follows the law of sine* 2 *azimuth.*

In each of the diagrams of Figure 53, the curve

Fig. 53.



represents the outline of the mass, and the magnetized needle at which it points is the compass-needle. The terrestrial energy is in the direction of the local dip, and the whole inductive energy will be in that

direction. Resolve this into horizontal and vertical directions. The effect of the vertical part will be, to produce a series of vertical linear magnets, each of which has its center at the same height as the compass-needle; and these produce no effect on the compass. The horizontal part remains, which is in the direction of the magnetic meridian, and is proportional to the horizontal force.

Now this horizontal induction does really produce a series of linear meridional magnets, as shewn in the first diagram: and the clothing of the surface will really be such as is shewn there. But we may resolve the induction into two, one parallel to the length of the mass as in the second diagram, and one transversal to that length as in the third diagram: and their energies will be respectively proportional to cosine azimuth of axis of mass, and sine of the same azimuth. The linear magnetic needles which they will produce, and the magnetic clothings, are shewn in the second and third diagrams. The aggregate of actions in the second diagram will be represented by that of one magnet, radial to the compass, whose entire action (as already said) is proportional to cosine azimuth: but the resolved part of this tending to give rotation to the needle receives the factor sine azimuth, so that its force tending to deflect the needle may be represented by

$$A \times \text{cosine azimuth} \times \text{sine azimuth}.$$

The aggregate of actions in the third diagram will be

represented by that of one magnet transversal to the radius, whose entire action is proportional to sine azimuth: but the resolved part of this tending to give rotation to the needle receives the factor cosine azimuth; so that its force tending to deflect the needle may be represented by

$$B \times \text{sine azimuth} \times \text{cosine azimuth}.$$

The total deflecting force is therefore

$$A \times \text{cosine azimuth} \times \text{sine azimuth}$$
$$+ B \times \text{sin azimuth} \times \text{cosine azimuth}$$
$$= \frac{A+B}{2} \times \text{sine 2 azimuth}.$$

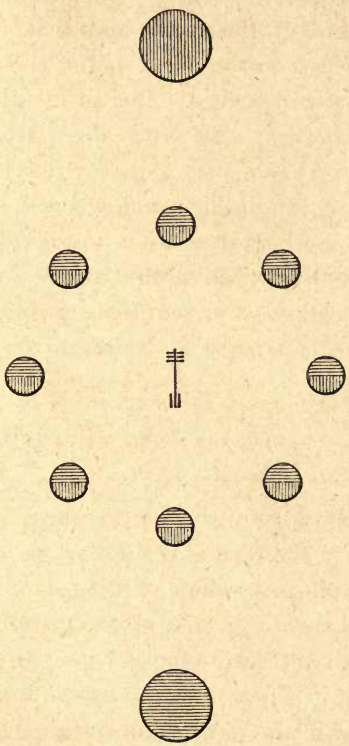
It is easy to see that the effects of the two parts of induction which we have considered have the same sign, and that the deflection produced in the compass-needle is such as to bring towards the mass of iron that pole which is nearest to the mass of iron.

This result agrees with the experiment which is described in Article 61.

66. *Simpler investigation when the mass is spherical with its center at the same height as the compass.*

In Figure 54, the sphere is represented in eight different positions, with the clothing of magnetism which is produced by the induction. In the northern and southern positions, the magnetism of that surface

of the sphere which is nearest to the compass-needle is of the kind opposite to that of the near pole of the



needle, and there is attraction between them: but this produces no deviation, because it is in the direction of the needle's length. In the east and west positions of the sphere, the magnetism of the north part of the sphere repels that of the north end of the needle, and the magnetism of the south part of the sphere repels

that of the south end of the needle, with equal forces, which balance: and in like manner there is equilibrium between the attraction of the north part of the sphere on the south end of the needle and that of the south part of the sphere on the north end of the needle: and the needle is not disturbed. But in all the other positions, the magnetism with which the nearest part of the sphere is charged is of such a quality that it attracts the nearest pole of the needle: and, when the sphere is in north-east or south-west position, the north end of the needle is made to deviate to the east: and, when the sphere is in north-west or south-east position, the north end of the needle is made to deviate to the west.

67. *In these cases, the magnitude of the deviation produced in the compass is independent of the magnitude of the terrestrial horizontal force.*

In order to judge of the law of compass-deviation in this case and in the case of the last article, as depending on the geographical position of the compass, that is, as depending on the magnitude of the terrestrial horizontal force (the only geographical element which affects this problem), it is necessary to observe that, the needle being directed in the magnetical meridian by the terrestrial horizontal force, and being made to deviate by a deviating force, the amount of deviation produced will depend upon the value of the fraction

$$\frac{\text{deviating force}}{\text{terrestrial horizontal force}}.$$

But, in a given position of a mass of iron, the deviat-

ing force depends only on the amount of magnetism produced by induction: and the amount of induction depends only on the terrestrial horizontal force which produces it: and therefore the deviating force is proportional to the terrestrial horizontal force: and therefore the fraction exhibited above is independent of the terrestrial horizontal force: and, in a given position of a mass of iron with respect to the compass, the deviation produced is the same in all parts of the earth.

68. *General investigation of the disturbance produced by a mass of iron symmetrical with respect to a vertical plane passing through the compass-axis* (*as an iron-built ship*) *subject to terrestrial induction.*

It is supposed here that, by the action of terrestrial magnetism, every particle of iron is converted into a small magnet whose direction is parallel to the local direction of the dipping-needle, and whose intensity is proportional to the local total intensity of terrestrial magnetism; the poles of the small magnet being in the same positions as those of the dipping-needle, or opposite to those of a magnet representing local terrestrial action. For convenience of language, we shall use terms applicable to a ship: but the results apply equally to any other masses of iron possessing the symmetry above-mentioned.

Let the center of the compass be the origin of co-ordinates; let A be the azimuth of the ship's head, measured from the magnetic north towards the east; a the azimuth of any particle measured from the ship's

head: so that $A+a$ is the azimuth of that particle from the north. Let b be the angular depression of the particle. Then if r be the distance of the particle from the compass; x, y, z, the ordinates towards the north, towards the east, and vertically downwards; we have

$$x = r \,.\, \cos b \,.\, \cos (A+a),$$
$$y = r \,.\, \cos b \,.\, \sin (A+a),$$
$$z = r \,.\, \sin b.$$

Let I represent the local intensity of terrestrial magnetism; δ the local dip, estimated positive for the northern hemisphere; m a constant for any particle under consideration, representing its susceptibility of inductive magnetization; $2l$ the length of the small magnet into which it is changed. Then the ordinates of the blue end of the small magnet are

$$x - l\cos\delta, \qquad y, \qquad z - l\sin\delta.$$

Its distance, or the square root of the sum of the squares of these quantities, omitting l^2, l^3, &c, is

$$r - \frac{l}{r}(x\cos\delta + z\sin\delta).$$

The resolved part of its attraction on the red end of the compass-needle in the direction of x is

$$Im\,(x - l\cos\delta)\left\{r - \frac{l}{r}(x\cos\delta + z\sin\delta)\right\}^{-3}$$
$$= Im\,\frac{x}{r^3}\left\{1 - l\frac{\cos\delta}{x} + 3l\,\frac{x\cos\delta + z\sin\delta}{r^2}\right\}.$$

Similarly, the attraction in the direction of y is

$$Im\,\frac{y}{r^3}\left\{1 + 3l\,\frac{x\cos\delta + z\sin\delta}{r^2}\right\}.$$

And the attraction in the direction of z is

$$Im\frac{z}{r^3}\left\{1 - l\frac{\sin\delta}{z} + 3l\frac{x\cos\delta + z\sin\delta}{r^2}\right\}.$$

(It is supposed here that the compass is so small that no sensible error will be produced in the small terms of these expressions, by adopting for the red end of the needle the values of x, y, z, which correctly apply to its center.)

The repulsions of the red end of the small magnet on the red end of the compass are the same, with no change but in the sign of l.

The true forces upon the red end of the needle, or the excesses of the attractions over the repulsions, putting H for terrestrial horizontal force or $I\cos\delta$, and V for terrestrial vertical force or $I\sin\delta$, are

$$\text{in } x \qquad -H.\frac{2lm}{r^3} + H.\frac{6lm.x^2}{r^5} + V.\frac{6lm.xz}{r^5}:$$

$$\text{in } y \qquad +H.\frac{6lm.xy}{r^5} + V.\frac{6lm.yz}{r^5}:$$

$$\text{in } z \qquad -V.\frac{2lm}{r^3} + H.\frac{6lm.xz}{r^5} + V.\frac{6lm.z^2}{r^5}.$$

These are the forces produced by a single particle upon the red pole of the compass-needle. To find the forces which all the iron of the entire ship produces upon that pole, we must take the sum of each of the factors of H or V through the whole ship. And for this

purpose we must so express these factors as to shew how much depends on the position of the ship's keel and how much on the position of the particle in the ship.

Now $x^2 = r^2 . \cos^2 b . \cos^2 (A + a)$

$= \frac{1}{2} r^2 \cos^2 b . \{1 + \cos 2A . \cos 2a - \sin 2A . \sin 2a\}.$

But, as a is the azimuth of the particle measured from the ship's head or from the line of the keel, there will be as many particles with a positive as with equal a negative. The term $\sin 2a$ will therefore vanish: and the sum of all the terms $\frac{6lmx^2}{r^5}$ will be (putting Σ to express the summation)

$$\Sigma \frac{3lm . \cos^2 b}{r^3} + \cos 2A . \Sigma \frac{3lm . \cos^2 b . \cos 2a}{r^3}.$$

Then $xz = r^2 . \sin b . \cos b . \cos (A + a)$

$= r^2 \sin b . \cos b . (\cos A . \cos a - \sin A . \sin a)$:

which in the same manner gives for the sum of $\frac{6lmxz}{r^5}$,

$$\cos A . \Sigma . \frac{6lm . \sin b . \cos b . \cos a}{r^3}.$$

And $xy = \frac{r^2}{2} \cos^2 b . \sin (2A + 2a)$

$= \frac{r^2}{2} \cos^2 b . (\sin 2A . \cos 2a + \cos 2A . \sin 2a)$:

which gives for the sum of $\frac{6lmxy}{r^5}$,

$$\sin 2A . \Sigma \frac{3lm . \cos^2 b . \cos 2a}{r^3}.$$

Also $yz = r^2 . \sin b . \cos b . \sin (A + a)$

$= r^2 . \sin b . \cos b . (\sin A . \cos a + \cos A . \sin a)$:

which gives for the sum of $\frac{6lmyz}{r^5}$,

$$\sin A . \Sigma \frac{6lm . \sin b . \cos b . \cos a}{r^3}.$$

Finally, $z^2 = r^2 . \sin^2 b,$

and the term $\frac{6lmz^2}{r^5}$ produces the sum

$$\Sigma \frac{6lm . \sin^2 b}{r^3}.$$

Now assume the following notation;

$$\Sigma \frac{2lm}{r^3} - \Sigma \frac{3lm . \cos^2 b}{r^3}, \text{ or } \Sigma \frac{2lm}{r^3} \left(1 - \frac{3}{2} \cos^2 b\right) = M,$$

$$\Sigma \frac{6lm . \sin b . \cos b . \cos a}{r^3} = N,$$

$$\Sigma \frac{3lm . \cos^2 b . \cos 2a}{r^3} = P,$$

$$\Sigma \frac{2lm}{r^3} - \Sigma \frac{6lm . \sin^2 b}{r^3} \text{ or } \Sigma \frac{2lm}{r^3} (1 - 3 \sin^2 b) = Q.$$

These four quantities $M, N, P, Q,$ do not depend on the terrestrial force or on the position of the ship, but are truly constants of the ship, depending only on its construction and its susceptibility of magnetism. Then the disturbing forces are,

*

In x or towards the magnetic north,

$$-HM+HP.\cos 2A+VN.\cos A.$$

In y or towards the magnetic east,

$$+HP.\sin 2A+VN.\sin A.$$

In z or vertically downwards,

$$-VQ+HN.\cos A.$$

These are the forces which act on the red end of the compass-needle. Those which act on the blue end are of the same magnitude but opposite signs, and therefore merely double the power which produces deviation of the needle.

69. *Examination of the physical meaning of the different terms of this disturbing force.*

First. If we compound together the terms $VN.\cos A$ towards the north, and $VN.\sin A$ towards the east, we find that they produce a term VN directed in the azimuth A, that is, directed to the head of the ship. This term therefore resembles in all respects a permanent magnetism of the ship, so long as the ship remains in one place. But it vanishes when V vanishes (that is, at the magnetic equator): and it changes sign when V changes sign (that is, in the south magnetic hemisphere): and this circumstance will give facility for determining the influence of this term, and correcting it by a magnet at each place of the ship. It will be seen from the expression for N that if the whole mass of iron is either

at the same level as the compass (making $\sin b = 0$), or below the compass (making $\cos b = 0$), the expression for VN vanishes.

Second. The terms $-HM + HP.\cos 2A$ shew that there is a term of fluctuating value in the meridional direction; if however P vanishes, that is if all the iron be below the compass, the fluctuation with change of azimuth vanishes. In any case, the force towards the north is affected *on the whole* by $-HM$: and when on the whole $\cos b$ is $< \sqrt{\frac{2}{3}}$, M is positive, and the terrestrial horizontal force is on the whole diminished.

Third. The term $HP.\sin 2A$ indicates a force changing its sign in every quadrant, which produces the quadrantal deviation described in Articles 61, 65, 66. It has no existence if the iron is entirely below the compass. It changes sign when $\cos 2a$ changes its sign: that is, when the iron is mainly at the sides of the compass.

The tangent of deviation actually produced will be $\frac{\text{deviating force}}{H}$. When V occurs as factor in the deviating force, the quotient $\frac{V}{H}$ is the same as tangent dip. When H occurs as factor, the deviation is independent of terrestrial force or dip.

The terms affecting the vertical force are of little interest in general. They would be comparable with observations only when the vertical force is accurately examined (as for instance by dip-observations).

70. *Defect of this theory: sketch of Poisson's more complete theory.*

We abstain, in this article, from expressing any doubt of the correctness of the assumption from which the preceding investigation starts, namely, that by the action of external magnetic force, every particle of the iron, or a limited proportion of the iron in every small space, is converted into a small magnet, whose axis is parallel to the axis of the external magnetism. The effect of such external magnetic force has been duly taken into account. But there is another disturbing magnetism not taken into account, namely, that every small magnet thus produced in the mass of iron produces a disturbing effect on every other small magnet.

We have reason to think, from the magnitude of that phænomenon of induction whose effects are most accurately known, namely the quadrantal disturbance, that the total effect of the internal action of the minute magnets is small in comparison with the effect of the terrestrial force: in some instances perhaps one-fifteenth part (with wide uncertainty). If, however, this be considered as a fair representation of its magnitude, then none of the preceding conclusions can be very wrong, and the theory of the last article may be accepted as sufficient for all practical cases. And it has this merit, that it shews clearly the dependence of the magnitude and sign of each force upon the distribution of the masses of iron.

Poisson undertook, in the *Mémoires de l'Institut de*

France, Tome v., the investigation of this theory in its most general form. He supposes that portions of the iron, occupying a space equal to $k \times$ entire volume of the iron (k being a fraction) are magnetic. In each of these spaces the two magnetic fluids can be separated to a constant distance in the direction in which the aggregate of external magnetism (both that of a distant body and that of the surrounding iron) acts on them, the amount of fluid so carried over to opposite sides being proportional to the intensity of the magnetism: this supposition amounts to exactly the same as that of the small magnets in the last articles. He then investigates symbolically (in a triple integral) the aggregate effect of all these little magnets upon any one: and, adding that aggregate to the external force, the total force upon that magnet and the direction of that total force are symbolically expressed. These must be the same as the force and direction assumed for it in the investigation; and the equations expressing this identity are the equations on the solution of which the solution of the problem depends.

It will be seen at once that this is a problem of very great complexity. In the general case, no advance whatever can be made to a solution of it. Only in the case of a sphere (solid or hollow), and a spheroid, can a result be obtained. One inference is, that the external attraction of the sphere is not much diminished by the hollow, until the hollow becomes so large as to leave a comparatively thin shell. For other purposes, it is conceived that a very oblate spheroid may in some

degree represent a sheet of metal, and that a very prolate spheriod may represent a rod. And these are the nearest approaches to practical application made by Poisson's theory.

The final statement by Poisson of general theoretical results is merely the following: If α, β, γ, are three rectangular components of terrestrial magnetic force, and if a system of iron masses receive magnetic induction from their action, the forces which they will exert on the pole of a compass-needle, in the direction of the three axes, will be

$$P\alpha + Q\beta + R\gamma,$$
$$P'\alpha + Q'\beta + R'\gamma,$$
$$P''\alpha + Q''\beta + R''\gamma,$$

P, Q, &c., being constants peculiar to the masses and positions of the iron. The reader who has seen in the preceding investigations how the resolved parts of terrestrial induction produce in the first instance three systems of molecular magnets, and who remarks that the mutual action of those in each system will still produce a series of magnets peculiar to that system, and who further remarks how the magnitude and direction of the resulting forces depend on the distance and position of each molecular magnet, and how the forces can be resolved into rectangular directions, will perceive that such a form of result is necessarily obtained without any abstruse investigation whatever.

71. *Inadmissibility of Poisson's fundamental sup-*

positions, and indication of the wants of a new theory.

It will be remarked in Poisson's theory (and also in the theory of Article 63) that it is assumed that the magnetism of both kinds originally attached to one molecule is never moved beyond the region of that molecule; and it is also implied in Poisson's assumptions, that the disturbance of this molecular magnetism is produced by the actions of other molecules or masses, without regard to the question whether those molecules or masses are in the same continuous substance. It follows from this that, according to the fundamental suppositions of both theories, if a given mass of iron be divided into any number of parts, the state of its magnetism and its action upon a compass-needle will not be altered. At the dividing planes there may be (theoretically) on one surface a peculia state of magnetism, but this will be accompanied by equal magnetism of the opposite kind on the other surface, and the proposition will still hold. The truth of the theories is easily tested by such experiments as the following:

Provide a bar of iron, 6 inches long, and also four bars whose section is the same, but each $1\frac{1}{2}$ inch long. Ascertain (by applying each end of each bar centrally to the *E* or *W* side of the compass) that they possess no permanent magnetism. Now apply the long bar endways at azimuth 45°, at any distance at which it will produce a quadrantal deviation of several degrees. Remove the long bar, and put the four

short bars in its place, just touching or merely separated by bits of thin paper: and the quadrantal deviation will be reduced to four-fifths of its previous value, or less. Or, apply the long bar sideways to the compass with its advanced end abreast of the center, and note the deviation; substitute the four short bars for it, and the deviation will be reduced to about three-fifths. The author has repeatedly tried these experiments, with bars of different lengths, and has always arrived at the same result.

The following observation is precisely similar, but on a much larger scale. It has been found by Capt. Evans, R.N., that when a ship is built of plates of iron very closely riveted together in every part, the quadrantal deviation of the compass is considerable. But, when a wood-built ship is covered with heavy iron armour, in plates which (though thick, and screwed to the wood, and perhaps lightly touching each other) are not riveted together, the quadrantal deviation is small.

In both these classes of experiment it is evident that Poisson's fundamental suppositions are at fault. It would seem that magnetism can flow through the unbroken bar or the closely-riveted iron nearly as through the steel of a magnet, but is not permanently retained as in a steel magnet. And it appears that; instead of a thin skin of magnetism on each side of the mass of iron, as in Article 65, Figure 53, where the thickness of the skin is determinate without reference to the depth of the iron; there will be a dense collection of magnetism of one or the other kind, brought from

the whole depth of the iron, and in some degree proportional to that depth. The results derived in Article 68 from the suppositions in Article 63, and the immediate results of Poisson's theory, are therefore both erroneous, in regard to the magnitude of the coefficients of the terms representing compass-disturbance. And we are left in complete doubt whether there is or is not a relation or system of relations between the coefficients in the general equations of Article 70. If, however, the inductional effects of external magnetic forces acting in the direction of different co-ordinates admit of being combined by the same law as the superpositions of small displacements, it would appear that formulæ similar to Poisson's will hold (with the doubt on relation of coefficients above mentioned), and that the law of quadrantal disturbance in Article 65 and of other disturbances in Article 68, will be unchanged.

These remarks lead us to a consideration of the really important defect in the present theory of magnetism of steel and iron. We possess no information, and no plausible theory, on the permanent distribution of magnetism in a steel magnet, or on the temporary distribution of magnetism in an iron bar affected by external magnetic action. It seems not unlikely that it may be subject to laws something like those of induced electricity. Many of the most important deductions can, however, be securely established without that knowledge; but, till it is obtained, we cannot regard magnetism as possessing the highest claims to regard as a Physical Science.

In the mean time, we prefer the theory of Article 63 to Poisson's theory; inasmuch as it gives distinct indications of the connexion between the arrangement of the masses of iron and the laws (irrespective of magnitude) of the compass-disturbance; depending so clearly on general principles of the translation of magnetism by the influence of external magnetic forces that they will never be materially modified. On these points, Poisson's generalities give no assistance.

72. *Complexity introduced, by induction, into the laws of the action of magnets upon each other.*

We may regard a magnet as consisting throughout of two materials: one the ferruginous part, of which the magnetism is liable to be shifted by the action of an external magnet: the other the magnetizable steel, properly so called, in which the magnetism is fixed. This produces (among other complications) a difference between the attractive and the repulsive powers. For, if the red pole of the first magnet is presented to the blue pole of the second, and strong magnetic attraction takes place, the red pole induces blue magnetism in the adjacent ferruginous part of the second magnet, and the blue pole induces red magnetism in the adjacent ferruginous part of the first magnet, and the total attraction is increased. But if the red pole of the first is presented to the red pole of the second, and there is consequent repulsion; each red pole induces blue magnetism in the adjacent ferruginous part of the other magnet, and

there is attraction, opposed to the magnetic repulsion: and the total repulsion is diminished. The same holds if the blue pole of the first is presented to the blue pole of the second.

The most accurate information which we possess on the subject is given by Mr W. Ellis, Assistant of the Royal Observatory, Greenwich, in a paper published in *The Philosophical Magazine*, May 1863, page 325. A magnet $5\frac{1}{2}$ inches long was attached to a clock-pendulum, and a similar magnet was so fixed in the clock-case that one pole of the swinging magnet passed over one pole of the fixed magnet: when there was attraction, the clock was accelerated; when there was repulsion, the clock was retarded; and both effects could be measured with extreme accuracy. The following results (extracted from a series) will shew the difference of the effects:

Distance between poles of magnets in inches.	Acceleration of clock in seconds per day when dissimilar poles were near.	Retardation of clock in seconds per day when similar poles were near.
0·05	+ 5·90	− 2·24
0·10	+ 4·10	− 2·12
0·20	+ 3·03	− 1·88
0·40	+ 2·05	− 1·42
0·80	+ 1·07	− 0·85
1·60	+ 0·34	− 0·29

and these may be thus divided;

Distance between poles.	Part due to permanent magnetism.	Part due to induced magnetism.
0·05	± 4·07	+ 1·83
0· 0	± 3·11	+ 0·99
0·20	± 2·455	+ 0·575
0·40	± 1·735	+ 0·315
0·80	± 0·96	+ 0·11
1·60	± 0·315	+ 0·025

It will be remarked that the part due to induced magnetism diminishes much more rapidly than that due to permanent magnetism.

73. *Method of measuring the amount of magnetism produced in a magnet by terrestrial induction.*

It is assumed in the following process that the law of distribution of induced magnetism, as regards the length of the magnet, is sensibly the same as that of permanent magnetism: and in fact the law at which we have arrived theoretically of induced magnetism, that the magnetism is exhibited as a coating at the end, is sensibly the same as that found experimentally for permanent magnetism, Article 12. The method adopted at the Kew Observatory for measure of the induced magnetism makes use of that part induced by the terrestrial vertical force, which in this magnetic latitude is large. As in Article 26, a deflexion-apparatus is to be used, in which it is made certain, in all stages of the operation, that the deflected needle (carrying a reversed telescope) occupies the same position with

regard to the viewing telescope; the whole apparatus being turned horizontally round a vertical axis till that condition is obtained, and the graduated horizontal circle which registers its rotation being then read. But there is this difference in adjustment from that of Article 26, that the magnet is placed in a vertical position, with a definite point near one pole exactly in the horizontal plane of the disturbed needle.

Suppose now that the red pole of the magnet is downwards, a mark near the red pole being at the same level as the needle, and the blue pole projecting far above the level of the needle. The effect of induction by the earth's vertical force is to add to the red power of the lower end and to the blue power of the upper end, and in fact to make the magnet more powerful. Now invert the magnet, so that the mark near the red pole is still at the same level as the needle, but the blue pole projects far below the level of the needle. As regards the action of the magnet upon the needle, the force exercised is the same as before. But to the red magnetism at the upper end of the magnet there is now added blue magnetism produced by the earth's vertical induction, and to the blue magnetism at the lower end there is added red magnetism produced by induction, and the power of the magnet is diminished. And these vertical magnetisms are not affected by the horizontal rotation of the apparatus round the needle. It is evident here that we have the means of determining the proportion of the induced part to the permanent part of magnetism.

In order to eliminate any conceivable excentricity in the location of the permanent magnetism, the operation may be repeated, using a mark near the blue pole of the magnet: and the mean of the two results may be taken. For simplicity of reduction, suppose that all observations are made with the separation between the centers of the magnet and the needle equal to the unit of measure, or (in England) 1 foot. As in Article 29, let A be the magnet-power of the magnet, and E the local horizontal magnet-power of the earth. Also let I be the magnet-power induced by the action of the terrestrial vertical force. In one position of the magnet, its power is $A+I$, and in the other position it is $A-I$. On account of the peculiarity of the magnet's position (which is different from those in Articles 26, &c.) these are not the powers which act in the present experiment: the real acting powers (see Article 53) will be found by multiplying those by an unknown constant e: so that the real acting powers in the experiment are $e\,(A+I)$ and $e\,(A-I)$. Let θ_1 and θ_2 be the deviations in the two experiments. Then as in the last sentence of Article 26, making $c=1$ and neglecting K (which which will have the same proportion in the two experiments),

$$e\,(A+I)=E\sin\theta_1;$$

$$e\,(A-I)=E\sin\theta_2;$$

whence

$$\frac{A+I}{A-I}=\frac{\sin\theta_1}{\sin\theta_2},$$

and $$\frac{I}{A} = \frac{\sin\theta_1 - \sin\theta_2}{\sin\theta_1 + \sin\theta_2} = \frac{\tan\frac{1}{2}(\theta_1 - \theta_2)}{\tan\frac{1}{2}(\theta_1 + \theta_2)}.$$

The value of I thus obtained is the amount of induced magnetic force produced by the local terrestrial vertical force. The local terrestrial horizontal force is found by multiplying the local vertical force by cotan. local dip: hence the induced magnet power produced by local terrestrial horizontal force E will be

$$A \times \text{cotan. local dip} \times \frac{\tan\frac{1}{2}(\theta_1 - \theta_2)}{\tan\frac{1}{2}(\theta_1 + \theta_2)}.$$

It is most carefully to be observed that the terrestrial force E and the dip here spoken of are *those peculiar to the place of this experiment;* that θ_1 and θ_2 are angles *peculiar to this experiment;* and that A is constant so long as the power of the magnet is not changed.

74. *Correction of the formulæ, used in the determination of the Earth's horizontal magnet-power, for effects of induction.*

In examining the process in Articles 26 to 28, it will be seen that we have to consider the effect of a magnet A in disturbing a needle when A is inclined to the meridian: and in Article 32, we have to treat of the earth's horizontal force upon A when A is in the meridian. These must be considered separately. Let E' be the earth's horizontal power at a second station.

First, for the disturbance of a needle by A. If ϕ be

the angle of deviation of the needle in a deflexion-experiment at the second station, the direction of the magnet A will be such, that its blue end will deviate by $90^\circ - \phi$ from the north meridian. The horizontal force which produces induction in A is $-E' \times \sin \phi$. Now we found that when the inducing force was E, the magnet-power produced by the induction was

$$A \times \text{cotan. original local dip} \times \frac{\tan \frac{1}{2}(\theta_1 - \theta_2)}{\tan \frac{1}{2}(\theta_1 + \theta_2)}.$$

Hence at the second station the induced magnet-power in the experiment of deviating the needle will be

$$-\frac{E' \sin \phi}{E} \times A \times \text{cotan. original local dip} \times \frac{\tan \frac{1}{2}(\theta_1 - \theta_2)}{\tan \frac{1}{2}(\theta_1 + \theta_2)},$$

or the efficient magnet-power will be

$$A \times \left\{1 - \frac{E' \sin \phi}{E} \times \text{cotan. original local dip} \times \frac{\tan \frac{1}{2}(\theta_1 - \theta_2)}{\tan \frac{1}{2}(\theta_1 + \theta_2)}\right\}.$$

But (see Article 27) $E' \sin \phi$ is sensibly equal to $2A$. Also, if θ be the deviation observed in the same manner at the original station, $E \sin \theta = 2A$. Hence $\frac{E' \sin \phi}{E} = \sin \theta$. And the efficient magnet power is

$$A \times \left\{1 - \sin \theta \times \text{cotan. original local dip} \times \frac{\tan \frac{1}{2}(\theta_1 - \theta_2)}{\tan \frac{1}{2}(\theta_1 + \theta_2)}\right\};$$

where, after once making the necessary experiments, the quantity within the bracket is constant for all stations.

This formula, it will be remarked, applies to the

efficient magnet-power of A at the second station when its distance from the needle is 1. At any other distance c, let the angle of deviation be ϕ'. Then the induced magnet-power will be

$$-\frac{E'.\sin\phi'}{E}\times A\times \text{cotan. original local dip}\times\frac{\tan\frac{1}{2}(\theta_1-\theta_2)}{\tan\frac{1}{2}(\theta_1+\theta_2)}.$$

But (see Article 27), putting 2 for m, E' for E, and ϕ' for ϕ,

$$E'.\sin\phi'=\frac{2A}{c^3}, \text{ and } \frac{E'\sin\phi'}{E}=\frac{1}{c^3}\cdot\frac{2A}{E}=\frac{1}{c^3}\sin\theta: \text{ and}$$

the total effective magnet-power of A will be

$$A\times\left\{1-\frac{1}{c^3}\sin\theta\times\text{cotan. original local dip}\times\frac{\tan\frac{1}{2}(\theta_1-\theta_2)}{\tan\frac{1}{2}(\theta_1+\theta_2)}\right\}.$$

Second, for the force producing the vibrations of A. The red pole of A will be turned to the north, and therefore the inducing force will be $+E'$; the induced magnet power will be $+\frac{E'}{E}.A\times$ cotan original local dip $\times\frac{\tan\frac{1}{2}(\theta_1-\theta_2)}{\tan\frac{1}{2}(\theta_1+\theta_2)}$: and the real power of the magnet will be

$$A\left\{1+\frac{E'}{E}\times\text{cotan. original local dip}\times\frac{\tan\frac{1}{2}(\theta_1-\theta_2)}{\tan\frac{1}{2}(\theta_1+\theta_2)}\right\}.$$

Here the factor of A does vary from station to station.

The reader will have little difficulty in investigating the corrections which these considerations shew to be necessary in the formulæ terminating in Article 32.

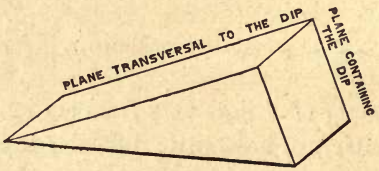
SECTION IX.

On Subpermanent Magnetism in Iron subjected to mechanical violence.

75. *Primary experiment on Subpermanent Magnetism: a long plate or slender bar of iron is placed on a firm frame (sometimes called the 'Magnetic Anvil'), with its length parallel to the local dip, and is struck repeatedly with a hammer: it becomes a magnet, with red magnetism in the end which dipped (in northern magnetic latitudes); and this magnetism does not change with change of the magnet's position.*

It will be convenient to prepare a small frame, as represented in Figure 55, of which the essential parts

Fig. 55.



are, one surface transversal to the direction of local dip,

and one surface containing the direction of local dip; and to fix it to the floor. Lay a piece of iron plate or iron bar, in contact with the dip plane, and with its length approximately in the direction of dip: and strike it repeatedly with an iron hammer. On removing it, it will be found to be a true magnet, the end which was lowest being charged with red magnetism: and this magnetism is not transient like the induced magnetism of soft iron, changing its place in the bar with every change in the position of the bar (see Article 60); but is constant like that of a steel magnet, retaining the same magnetism whatever be the position of the bar. An iron bar which has not been struck, if applied in the horizontal position end-on towards the center of a compass, does not disturb the needle at all; but the same iron bar when it has been struck in the manner described, if applied end-on to the center of a compass, disturbs it powerfully: one end deflecting the needle in one direction, and the other end deflecting the needle in the opposite direction, exactly as a steel magnet would do it: and in all respects comporting itself as a steel magnet.

76. *Variations of the experiment. All lead to the supposition that iron, in a state of tremor or jar, is peculiarly able to receive induced magnetism and to retain it firmly.*

The circumstances of the last experiment, in which the receipt of magnetism depends evidently on the

placing the iron in that position in which the earth's power acts most strongly on it, suggest the trial of a steel magnet for the same purpose. Lay a steel magnet E and W on a table, and bring a nail near it in the same direction, not touching the magnet. In this state, the earth's magnetism has no effect: and the nail receives from the magnet no permanent magnetism (as is easily verified by applying it end-on to a compass). Now place the nail in its former position, and strike it with a hammer; upon withdrawing it, it will be found that it has become a magnet in all respects like a steel magnet, the pole which was nearest to the large magnet having magnetism opposite to that of the pole which it approached.

If a bar of iron be dropped endwise upon a stone pavement, it immediately acquires polar magnetism of permanent character, like that of a steel magnet. But if it be dropped on a carpeted floor, it scarcely receives any sensible magnetism. This experiment shews that a state of sharp tremor or violent jar among the particles of the iron is necessary to enable it to receive this magnetism.

As matter of familiar observation it may be mentioned that a common fire-place poker, of which the same end is usually downwards and is frequently struck upon the hard floor, is almost always well charged with magnetism, its red end being the lower.

77. *Reversion or destruction of the magnetism. Origin of the term 'Subpermanent.'*

Suppose that the lower end of the bar in the experiment of Article 75 is distinguished by being painted white. This white end, after the bar has been placed with its white end in the direction of dip and has been struck, is found to be charged with red magnetism. Now reverse the bar upon the dip plane with its white end upwards, and strike it; it will be found that its white end is charged with blue magnetism. The magnetism has been reversed. Undoubtedly, in order to arrive at this state, it has gone through the stage of being destroyed or rendered undiscoverable by instruments.

But the pure destruction may be visibly effected in the following manner. Lay the bar upon that surface of the magnetic anvil which is normal to the direction of dip, and strike it with a few blows of the hammer. On removing the bar, and testing its state by means of a compass, it will be found that all trace of magnetism has disappeared. The bar is now in the same state as before the experiment of Article 75.

If, however, the magnetized bar be subject to no such violence, but be suffered to rest quietly, or be moved gently into different positions, it will slowly lose a large proportion of its magnetism. And it is this peculiar character which necessitated the introduction of a new name. The magnetism of a struck iron bar resembles the magnetism of a permanent

steel magnet in all respects but this, that, while perhaps no change can be remarked in hours or days, it infallibly diminishes in a long time. To express this partially permanent character, the term 'Subpermanent Magnetism' has been adopted.

In single bars, the subpermanent magnetism diminishes sensibly in a few hours, and is lost in a few days. In some large iron ships, a portion of it has remained unaltered for many years. It would seem that where the operation of magnetizing by hammer-blows has been rapid, the magnetism is not very firmly fixed: but where the violence has been long continued, the magnetism is so firmly established as to become an immovable quality of the iron.

SECTION X.

On the Magnetism of Iron Ships, as affecting their Compasses.

78. *Philosophical and Commercial Importance of this subject. Complication of the Magnetic considerations involved in it.*

It is unnecessary to remark on the extent to which at the present time, when so large a portion even of the mercantile navies is built of iron, the interests of commerce are involved in the investigations which alone can make the ship-compasses available. But it may be desirable to point out to what an extent Science may benefit from it. It will be shewn that the principal agent in the disturbance of the compass is subpermanent magnetism, an element little known before the introduction of iron ships, and whose laws have principally been derived from the examination of iron ships. But another element, whose effects are sensible in all, and very important in some, is transient

induced magnetism: and the study which in late years has been given to this subject has been stimulated almost entirely by its application to iron ships.

In applying the science deductively to the control of ships' compasses, every part of the theories treated in the earlier sections is brought into play. Magnetic declination is obviously necessary: terrestrial horizontal force enters into every formula of disturbance (see Article 55): both horizontal force and vertical force, or dip, occur in the formulæ for induction disturbance: the laws of action of magnet-power enter both into the effects of subpermanent magnetism of the ship and into those of the permanent magnets employed in correction: and the theory of induced magnetism, and especially of quadrantal deviation, presents itself in the correction of the effects of the iron masses.

79. *Brief history of the first steps in this science.*

The first real step appears to have been made by Captain Flinders, about 1803, who remarked that the disturbances of his compass were such as would be produced by the attraction of iron charged with magnetism; blue for northern latitudes and red for southern latitudes, in the direction of the ship's head; and suggested the use of a vertical bar to be placed aft of the compass, whose upper end having similar magnetism would tend to correct the other. At a later time, 1820 to 1833, numerous experiments were made by Professors Barlow and Christie, illustrating the action of induced

magnetism. In regard to subpermanent magnetism, the first experiments on iron bars, &c., were made by Mr Scoresby, about 1821; and the first virtual observation in ships was made by General Sabine in discussing the compass-deviations in Sir James Ross's voyage, 1839 to 1843, in which he remarked that the peculiarities in the disturbances of the compass lasted for a short time after the ship had left the region in which the terrestrial forces were such as would tend to explain the disturbances. These observations were made in wood-built ships having many accidental masses of iron.

The first explanation of the character of the compass-disturbance produced by iron ships was given by the writer of this Treatise in the *Phil. Trans.*, 1839, as resulting principally from examination of the iron steamer, *Rainbow*, in 1838. The disturbing forces on the steering-compass of that ship were so great that in one position of the ship the north end of the needle was deflected more than 50° to the east, and in another position it was more than 50° to the west. The first light that was thrown upon the causes of these deviations was obtained by placing the ship with her head exactly north (which can be done in various ways, one of the most convenient being to use an azimuth-compass on shore, and to adjust the ship by signal till her masts, as seen by the shore-compass, are all in the magnetic meridian), then observing the deviation of the compass, and, replacing the compass by a vibrating needle whose time of vibration on shore

had been found, ascertaining the effective total horizontal force acting on the compass needle; this total force, which is really in the deviated direction of the compass, was resolved into a N. direction and an E. direction, and gave the whole force to the N. and that to the E.; subtracting the terrestrial force from the former, the ship's disturbing forces to the N. and to the E. were found. Similar operations were performed with the ship's head E., S., and W.; and in each, the amount of disturbing force in the direction of ship's head and ship's starboard-side were found. On examining these it was at once seen that the four values for force directed to the ship's head agreed so nearly, and the four values to the ship's side agreed so nearly, as to leave no doubt that nearly the whole disturbing force was a force similar to permanent magnetism making a definite angle with the ship's keel. Then, deductively, using the means of these values, and compounding them with terrestrial horizontal force in different positions of the ship's head, on the principles of Article 55, the resulting direction of the needle in 33 positions of the ship was found to agree with the observed direction; only a small quadrantal difference remained, which was evidently explained by the quadrantal term in Article 68, as produced by masses of iron towards the ship's head.

Similar treatment of observations on three other compasses in different parts of the ship gave nearly the same value for the permanent magnetism transversal to the keel, but smaller values for the longi-

tudinal values. It was certain therefore that there was permanent or subpermanent magnetism transversal to the keel (because no induction could account for it); but it was difficult to say how much of the longitudinal part was subpermanent, how much due to the first term in Article 69, and how much to the iron stern-post acting as a vertical magnet (Articles 60 and 53).

The laws thus ascertained were verified by placing below the compass a magnet, in the position opposite to the ship's magnetism, and at a distance which (as had been ascertained by experiment) would enable the magnet to produce an effect equal to the ship's effect; and also applying a mass of iron at one side of the compass to correct the quadrantal deviation (Article 62). Then, in swinging the ship round, the compass was found to be correct in every position of the ship.

In the next experiments, a change of great value was made in the practical operations; founded on the following theoretical considerations. Conceive the ship's magnetism to be resolved into two parts, one transversal to the ship, one longitudinal. When the ship's head is placed north or south, the transversal force alone disturbs the compass, and the quadrantal disturbance vanishes (Articles 65 and 66); and the transversal magnetic part can be corrected by an opposite transversal magnet broadside-on to the compass, whose distance is determined without any calculation, simply by trying its effect at different distances till the needle points

correctly. Then, in like manner, if the ship's head is placed east or west, the longitudinal magnetism only disturbs the compass, as the quadrantal deviation vanishes there, and it is to be corrected by a longitudinal magnet broadside-on to the compass, tentatively applied. The effects of permanent or subpermanent magnetism are now entirely corrected. In order to correct for the induction-effect which produces quadrantal deviation, the ship's head must be placed in azimuth 45° (nearly), or 135°, or 225°, or 315°; there will be no difficulty in ascertaining whether the quadrantal disturbance is such as corresponds to the effect of iron in the direction of the ship's head: and, if so, it must be corrected by iron on one or both sides, shifted by trial till the correction is complete.

These processes were introduced by the author in 1838, and they are still retained in use without alteration.

80. *Reference to the causes of partial failure in the correction of the compass.*

So long as the ship's magnetism remains unaltered, and so long as she remains in the same region of the earth, her compass will now be perfectly correct. But it is necessary to examine the changes which take place in lapse of time and in change of geographical position.

The determining circumstances of a ship's subpermanent magnetism, and the effect of time upon it, including the effect of changing the position of the

ship's head while in the same locality, and subjecting her to some degree of violence, were first investigated a few years after the establishment of the principles above described, by a 'Liverpool Committee' appointed by merchants of the City of Liverpool. From their researches it clearly appeared that the magnitude and direction of a ship's subpermanent magnetism are connected with the direction in which she is built, exactly according to the law explained in Article 75; and thus no doubt remained that the subpermanent magnetism was the effect of terrestrial magnetism acting on the iron during the heavy hammering to which an iron ship is subjected in the operations of building. And it was found that if, after launching, the ship is kept for a time in a different position, and more especially if she is subject to the tremor of steam-navigation, a large proportion of this subpermanent magnetism vanishes: a part however remaining invariable. Now the effect of either making no attempt to correct the subpermanent magnetism by magnets, or of correcting it completely and then finding that, in consequence of the decay of the subpermanent magnetism, it is now overcorrected, may be estimated by reference to the construction of Article 55, Figure 45. It is there seen that, with a given change of ship's magnetism, the error produced in the needle's direction will depend on the magnitude of the local terrestrial horizontal force: if the ship is in a part of the earth where horizontal force is large, or where BE is large, the error of the compass will be small: but if the horizontal force is

small, the error of the compass will be large. To meet the effects of this time-change it is very desirable that the magnets by which the compass is corrected should be so mounted that they can at any time be easily adjusted to the distance at which they make the compass correct. Gray's Binnacle is arranged expressly for this purpose.

The effect of change of geographical position is sometimes very troublesome. If the subpermanent magnetism is not completely neutralized by the magnets, the remaining error, as above explained, produces different deviations in different localities where the horizontal force is different, and no table of errors which applies to one place will apply to another place. In the formulæ of Article 68, and their explanation in Article 69, it will be seen that induction produces a term VN representing magnetism directed towards the ship's head: and it is impossible to say, in the operations for correcting the compass, whether this term has a real value: and the compass must be corrected as if it did not exist. But when the ship's magnetic latitude is changed the value of V is changed; and when she goes into the south magnetic hemisphere where the dip is reversed (Figures 21 and 37), the sign of V is changed; and these changes in the value of VN produce a great change in the value of longitudinal magnetism which was corrected as if it were constant; the correction therefore is no longer valid, and a considerable error is produced in the compass. But upon looking at the expression for N in Article 68, in which

the factor $\cos a$ is negative for iron which is astern of the compass, it will be seen that; though N may be large in a merchant-ship, where the compass is placed very near the stern; yet it will be small in a ship of war where the compass is much nearer the center of the ship.

There is another serious cause of error. The vertical stern-post and rudder-post become in fact, by induction, vertical magnets: but the upper end which acts on the compass is charged with blue magnetism in the northern hemisphere, and red magnetism in the southern hemisphere. This change, however, may be neutralized by adopting a suggestion of Mr. Rundell, that another vertical bar be fixed in the ship on the opposite side of the compass. In ships of the Royal Navy, the compass is too distant from the stern-post to be sensibly affected.

When due attention is given to these considerations, and the ship is not very new, it has been shewn by the writer, in the *Phil. Trans.*, 1855, from examination of the disturbances in several ships, that, with insignificant alteration in the position of the magnets, a ship's compass will be perfectly correct in all parts of the world.

81. *Continuation of the history. Investigation of the effects of the ship's heeling.*

In the Admiralty *Manual of the Compass*, and in other papers, Mr. Archibald Smith has elaborately discussed the forces acting on the compass. Those

which depend on subpermanent magnetism require no special notice: but those depending on induced magnetism are founded exclusively on Poisson's general equations given at the end of Article 70. These are the last attempts made on formation of a theory: though discussions of the magnetic phænomena presented by special ships, leading to conclusions of great interest, have been published by the same writer and by Capt. Evans, in the *Philosophical Transactions*.

A very important field for the application of theory is afforded by the consideration of a ship's 'heeling' or inclination to one side. Usually, in an iron ship, when her head is placed *N.* or *S.*, the ship's inclination through an angle of n degrees disturbs the compass through an angle of about n degrees; but in some particular instances, it has been known to disturb the compass as much as $2n$ degrees. This effect is very serious in those parts of the earth where the wind is steady and the ship is inclined in the same direction for many days or weeks in succession. All the preceding investigations have gone on the supposition that the ship was 'on even beam,' or that her deck was horizontal. The investigations for the inclined position are necessarily very complicated. We shall give an outline of them, as they proceed in sequence from those obtained for the horizontal position of the deck in Article 68.

Let h be the angle of the ship's heel towards the starboard side: we shall suppose that, in the induced forces, the square of h may be neglected. Also let

F be the subpermanent force acting on the red end of the needle towards the fore part of the ship: S that force parallel to the ship's deck which acts towards the starboard side: K that towards the ship's keel. When the ship heels through the angle h, the forward force is not thereby disturbed, but the horizontal force towards the starboard side becomes $S\cos h - K\sin h$. But the transversal magnet which has been correcting the compass when the ship was on even beam, and whose power therefore was $-S$ in the plane parallel to the deck, now exerts the horizontal power $-S\cos h$. Therefore the new force of subpermanent magnetism in the horizontal plane, towards the starboard side, produced by the ship's heeling is

$$-K\sin h,$$

which produces

$$+\sin h.K.\sin A \text{ towards the north,}$$

$$\text{and } -\sin h.K.\cos A \text{ towards the east.}$$

For the forces produced by induction, let x, y, z, be measured horizontally towards the north, horizontally towards the east, and vertically, as in Article 68. Consider $\cos h$ as $= 1$. Then, using the notation of Article 68,

$$x = r.\cos b.\cos a.\cos A - r\cos b.\sin a.\sin A + \sin h.r.\sin b.\sin A;$$

$$y = r.\cos b.\cos a.\sin A + r.\cos b.\sin a.\cos A - \sin h.r.\sin b.\cos A;$$

$$z = r.\sin b + \sin h.r.\cos b.\sin a.$$

The new term introduced into x^2 is

$$\sin h.2r^2\{\sin b.\cos b.\cos a.\sin A.\cos A-\sin b.\cos b.\sin a.\sin^2 A\},$$

and therefore the new term introduced into $H.\Sigma\frac{6lmx^2}{r^5}$ will be

$$\sin h\,.\,H\,.\sin 2A\,.\,\Sigma\frac{6lm\,.\,\sin b\,.\,\cos b\,.\,\cos a}{r^3}$$

$$=\sin h\,.\,HN\,.\,\sin 2A.$$

The new term introduced into xz is

$$\sin h.r^2\{\cos^2 b.\sin a.\cos a.\cos A-\cos^2 b.\sin^2 a.\sin A+\sin^2 b.\sin A\}$$

and therefore the new term introduced into $V.\Sigma\frac{6lm.xz}{r^5}$ will be

$$\sin h\,.\,V\,.\sin A\,.\,\Sigma\,\frac{6lm\,(\sin^2 b-\cos^2 b\,.\,\sin^2 a)}{r^3}.$$

Call the summed fraction R: (it will be seen that $R=M+P-Q$): then the term in question becomes

$$\sin h.\ VR\,.\sin A.$$

Combining this with the preceding, the whole additional term towards the magnetic north

$$=\sin h\,\{HN\,.\sin 2A+VR\,.\sin A\}.$$

The new term introduced into xy is

$$\sin h\,.\,r^2\,\{-\sin b\,.\cos b\,.\cos a\,(\cos^2 A-\sin^2 A)$$

$$+2\sin b\,.\cos b\,.\sin a\,.\sin A\,.\cos A\},$$

and therefore the new term introduced into $H.\Sigma\frac{6lm.xy}{r^5}$ will be

$$-\sin h.H.\cos 2A.\Sigma\frac{6lm.\sin b.\cos b.\cos a}{r^3}$$

$$=-\sin h.HN.\cos 2A.$$

The new term introduced into yz is

$$\sin h.r^2\{\cos^2 b.\sin a.\cos a.\sin A+\cos^2 b.\sin^2 a.\cos A-\sin^2 b.\cos A\},$$

and therefore the new term introduced into $V.\Sigma\frac{6lm.yz}{r^5}$ will be

$$\sin h.V.\cos A.\Sigma\frac{6lm(\cos^2 b.\sin^2 a-\sin^2 b)}{r^3}$$

$$=-\sin h.VR.\cos A.$$

Combining this with the preceding, the whole additional force towards the magnetic east is

$$-\sin h\{HN.\cos 2A+VR.\cos A\}.$$

Uniting the terms derived from subpermanent magnetism and from induction, we have the following forces introduced by the ship's heeling:

Towards the north,

$$\sin h\times\{K.\sin A+HN.\sin 2A+VR.\sin A\}.$$

Towards the east,

$$-\sin h\times\{K.\cos A+HN.\cos 2A+VR.\cos A\}.$$

The latter is the only force which disturbs the direction of the compass-needle.

82. *Examination of the heeling-disturbance, and remarks on the possibility of correcting it.*

The quotient of the deviating force by the terrestrial directive force, on which the needle's deviation will depend, will be (remarking that $\frac{V}{H} = \tan \text{dip}$)

$$-\sin h \times \left\{ N . \cos 2A + \left(R \tan \text{dip} + \frac{K}{H} \right) \cos A \right\} .$$

No simple rule can be given for the position of the ship's head which will make the bracket vanish: cos A will be determined by a quadratic equation.

The first term has for factor N. Now in examining Articles 68 and 69, it will be seen that N is that effect of induction which puts on the appearance of a constant magnetic force parallel to the ship's keel. The correction of N by a magnet is of no avail in reference to the formation of the first term in the last article. But correction of N by a mass of iron subject to the same induction as the rest would destroy the term in the last Article. In the ordinary place of the steering-compass in a merchant-ship, it may happen that this term is negative and large, principally as affected by the magnetism of the sternpost: and the treatment of the heeling error is very unmanageable. There appears to be no way of determining the value of the bracket in different azimuths, except by inclining the ship in different azimuths. Here we see a great advantage in the use of Mr. Rundell's vertical bar in front of the compass. This,

which is subject to induction, if so adjusted as to correct N when the deck is level, will also correct N in the heeling term: and the part depending on cos $2A$ will disappear. In ships where the steering compass is much nearer the middle of the ship, N will usually be small.

Supposing then that N is put out of consideration, the term that remains is

$$-\sin h \times \left(R \tan \text{dip} + \frac{K}{H}\right) \cos A.$$

Both terms of the bracket become large in high magnetic latitudes, where the dip is large and H is small. If K be positive, or tending to draw the red end downwards (as will hold in the subpermanent magnetism produced by the operations in the process of building iron ships in north latitudes), the second term, which is the larger, will be negative; and, remarking the sign of cos A, when the ship's head is north of the east-and-west-line, and the ship heels to starboard, the red end of the needle will be drawn to the west: when the ship's head is southerly of that line, the red end will be drawn to the east. Both cases are included in the seaman's rule "the red end of needle deviates to the windward side." In southern magnetic latitudes, it is the blue end which so deviates.

When the ship's head is east or west, that is, when $A = 90°$ or $270°$, the heeling force vanishes: it is maximum, with different signs, when the ship's head is north or south, that is, when $A = 0°$ or $= 180°$.

The circumstance that the deviating force is expressed

by a multiple of $\sin h \times \cos A$ enables us to correct it by application of a magnet. In Article 51, Figure 41, putting h for ϕ, that is to say placing a magnet in the ship which shall be vertical when the ship is on even beam and which will have the inclination h when the ship has the heel h, we found that its horizontal force on the red end of the needle is

$$4\alpha \frac{ca}{(c^2-a^2)^2} \times \sin h.$$

When the ship's head is north, or $A = 0$, this force acts transversely to the needle, and is wholly available (and so, with changed sign, when the ship's head is south). But in any other position of the ship's head, the force acts obliquely on the needle, and must have the factor $\cos A$. The efficient force is therefore

$$4\alpha \times \frac{ca}{(c^2-a^2)^2} . \sin h . \cos A.$$

This follows the same law as the force which we wish to neutralize: and therefore, by proper choice of the poles of the magnet, and by sliding it up and down parallel to the ship's masts, a position may be found in which it will entirely correct the heeling-error.

Unfortunately, the terms included in the bracket both depend on geographical position, and the correction which is valid in one part of the earth will not be valid in other parts. The correction of the heeling-error deserves, more than any other point, the attention of practical magnetists.

SECTION XI.

On the Continuous Registration of Small Changes in Terrestrial Magnetism.

83. *General principle of photographic self-registration now usually adopted. Distinction of the magnetic elements which are to be registered, and appropriate positions of the recording apparatus.*

The object to be attained is, to make an impression depending on the position of some part of the apparatus, without contact, or friction, or mechanical resistance of any kind. Nothing is so suitable for this purpose as photography. If from a minute source of light (as a lamp shining through a very small aperture) light falls upon a concave mirror, or upon a plane mirror assisted by a convex lens, which is firmly attached to a moving part of the apparatus; then a spot of light (the optical image of the small source of light) may be formed at a proper distance, and the motions of the moving part of the apparatus will produce corresponding motions of the

spot of light; which, if received on photographic paper, may be made to impress a permanent register of the position of the spot, from which the positions of the moving apparatus may be inferred.

It is now necessary to explain how the time is registered in combination with the register of the spot-movement. For this purpose, the photographic paper must be attached, either to a plane board which is moved by clock-work uniformly in its plane in the direction at right angles to that in which the motions of the spot occur, or to a barrel which is made to rotate uniformly and whose axis is parallel to the motions of the spot. With either of these, the motions of the spot leave on the paper a photographic curve, whose abscissa represents time at a given length for an hour, and whose ordinate represents a quantity proportional to the instrumental movement which causes the motion of the spot. If we interrupt for a short time the beam of light (which will cause an interruption in the photographic curve), noting also the clock-time, we can mark off accurately the hours, &c., on the time-scale. And if we possess any independent methods of observing the position of the moving apparatus at definite times, we can, by adjusting the scale of ordinates to the spot-position at those times, make it available for every other time.

The elements which most conveniently represent the state of terrestrial magnetism as acting at any one geographical point, and whose changes it is desirable to record, are,—the position of the free magnet, the small changes

of which may be conceived as the effect of a westerly magnetic force acting on the red end—the magnitude of the horizontal directive force H—and the magnitude of the vertical force V. For numerical expression of all the small changes of force, it is convenient to use H as the unit. The changes of declination of the free magnet are in the horizontal plane, and therefore the axis of the barrel on which they are registered ought to be horizontal: a horizontal position also, it will be shewn, can be made available for register of the changes of H: but for the changes of V it will usually be found convenient, though not absolutely necessary, that the axis of the barrel be vertical.

84. *Record of the small changes of magnetic declination, and evaluation of their scale.*

The apparatus, as will be gathered from the last article, is exceedingly simple: a fixed source of light; a concave mirror, firmly connected with the frame that carries the magnet, and causing the pencil of light to converge to a spot; and the revolving barrel with horizontal axis which receives that spot. Suppose now that the distance of the concave mirror from the surface of the barrel where the spot is formed is m inches. To give to the spot a motion of 1 inch, the beam of reflected light must have been turned through the angle $\frac{1}{m}$: and therefore, as the direction of the incident light is invariable, the mirror (and the magnet which accompanies

it in its motions) must have turned through the angle $\frac{1}{2m}$. The direction of horizontal magnetic force has therefore changed through an angle represented by $\frac{1}{2m}$. This change of direction would be produced by combining, with the northerly directive force, a westerly force equal to

$$\frac{\text{northerly directive force}}{2m}.$$

Consequently a motion of the photographic spot through 1 inch in the direction of the ordinate of the curve will represent a westerly magnetic force equal to $\frac{1}{2m}$ of the whole northerly horizontal force.

85. *Bifilar magnetometer for record of the small changes of magnetic horizontal force, and evaluation of their scale.*

A torsion-apparatus of any kind, which permits an accurate measure to be made of the force that produces any angle of torsion, would answer perfectly for this object. But the kind of torsion which has been adopted as most convenient is that produced by suspension by means of two cords or wires, separated, both at the upper place of attachment to a fixed beam or other support, and at the lower place of attachment to the magnet.

Suppose an unmagnetic bar (as of brass) to be suspended thus by two cords separated at the top and at the bottom. The bar will take such a position that the two cords will hang in one vertical plane. Let the apparatus be so adjusted that the unmagnetic bar takes a position in the magnetic meridian. Substitute for it a magnetized steel bar; the steel magnet is in the position which it would assume if perfectly free, and therefore it exerts no mechanical effort to escape from that position. Now turn, through a limited angle in the horizontal plane, the substance to which the upper ends of the two cords are attached. The two cords are now no longer in one plane: and they exert a force of torsion or wringing on the suspended magnet. The magnet will yield to this, but not entirely; for, as soon as its position makes an angle with the magnetic meridian, the earth's directive force tends to pull it back towards the magnetic meridian, or to resist the torsion-power produced by the bifilar suspension. The magnet therefore will take a position in which the torsion-power, produced by the circumstance that the two wires are not in one plane, exactly balances the torsion-power produced by the action of terrestrial directive force upon the magnet, now inclined to the magnetic meridian.

Now suppose the terrestrial directive force suddenly to increase. It will more than balance the torsion-power of the suspension, and will draw the magnet nearer to the meridian. Suppose the terrestrial directive force to diminish: the torsion-power of the

suspension will overcome it, and will turn the magnet further from the magnetic meridian, till the balance is restored. It is plain here that, by noting the position of the magnet, we have the means of ascertaining the direction and magnitude of the changes which terrestrial directive force undergoes.

It is indifferent whether the rotating apparatus (or 'torsion circle') be connected with the fixed beam, so as to act on the two upper points of attachment, or with the magnet, so as to act on the two lower points of attachment. It is also indifferent whether, in the former case, the two lower points are in the longitudinal direction of the magnet: in the three diagrams to which we shall now refer, we shall suppose that they are in an inclined position.

Figure 56 is a side view of the magnet in an

Fig. 56.

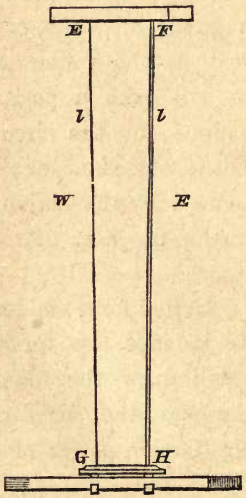
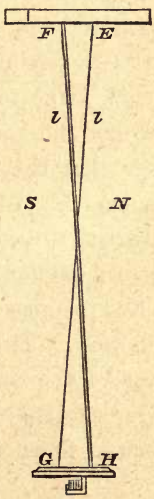
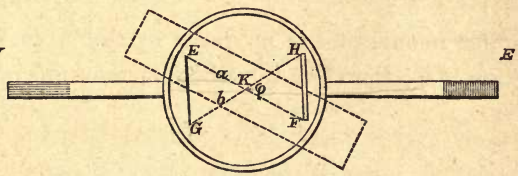


Fig. 57.



assumed position of the points of attachment: Figure 57 is an end view. These views shew that the two wires are not in one plane: (the angle of crossing is very much exaggerated in the diagrams.) Figure 58

Fig. 58.



represents the view from above, or the projection of the whole upon a horizontal plane: this will give the means of computing the torsion-strain produced by the weight of the magnet.

Let the distance EF of the upper points of attachment be $2a$, and the distance GH of the lower points be $2b$: and let them make the angle ϕ: also, let the length of each suspension-wire be l; and the weight of the magnet W. The torsion of each cord will be sensibly $\frac{W}{2}$; and the resolved part of this in the direction EG, Figure 58, will be $\frac{EG \times W}{2l}$; and the momentum of this to turn the magnet will be

$$\frac{W}{2l} \times EG \times KL,$$

KL being the perpendicular from K upon EG. But

$EG \times KL = 2$ area of triangle $EKG = ab \,.\, \sin\phi$: therefore the momentum of the strain in the direction EG to produce rotation of the magnet is

$$\frac{W}{2l} ab \,.\, \sin\phi.$$

A similar momentum is produced by the strain in the direction FH: therefore the whole momentum of rotation is

$$\frac{W.ab}{l} \sin\phi.$$

Now let the upper suspension-bar be turned round till the magnet is turned to a position at right angles to the magnetic meridian. The momentum of terrestrial horizontal magnetism upon it, by Article 21, supposing it inclined to the magnetic meridian by the angle θ, will be $E \,.\, B \,.\, \sin\theta$: and $\sin\theta$ will sensibly $= 1$, not only when $\theta = 90°$, but also when $\theta = 90° + x$, where x is a small angle (such as we have to consider) which makes $\sin\theta = 1 - \frac{x^2}{2} +$ &c. We shall therefore consider the momentum of terrestrial horizontal magnetism as $= E \,.\, B$. And, as this balances the momentum of torsion, we have the equation

$$\frac{W.ab}{l} \sin\phi = E \,.\, B.$$

Now conceive E to be variable, and ϕ to vary in consequence. The equation of variation is

$$\frac{W.ab}{l}\cos\phi.\delta\phi = B.\delta E.$$

Dividing this equation by the last,

$$\text{cotan}\,\phi.\delta\phi = \frac{\delta E}{E}.$$

Thus the ratio in which the Earth's horizontal magnetic force (E or H) varies is inferred at once from $\delta\phi$; that is, from the angular change, in the horizontal plane, of the position of the magnet. To make this measurable, let a concave mirror, or a plane mirror assisted by a lens, be attached to the magnet so as to partake of its angular vibration in the horizontal plane: let light from a fixed lamp fall on it; and let it form an image of the light upon a rotating barrel covered with photographic paper at distance n inches. A motion of the spot through 1 inch corresponds to an angle in the position of the reflected ray represented by $\frac{1}{n}$, and therefore to an angle in the position of the mirror represented by $\frac{1}{2n}$; therefore, for a motion 1 inch in the spot, $\delta\phi = \frac{1}{2n}$, and

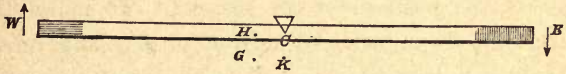
$$\frac{\delta E}{E} \text{ or } \frac{\delta H}{H} = \frac{1}{2n} \times \text{cotan}\,\phi.$$

This is the change of terrestrial horizontal force corresponding to a motion of 1 inch in the photographic spot: by means of this value, a general scale for interpreting the values of the spot-motion on the ordinates of the photographic curve can be formed.

86. *Balance-magnetometer for record of the small changes of magnetic vertical force, and evaluation of their scale.*

Let Figure 59 represent the balance-magnetometer:

Fig. 59.



a magnet to which is attached a steel knife-edge *C*, by means of which the magnet vibrates in the vertical plane; its knife-edge being supported by horizontal planes of hard stone. The vertical plane being transversal to the magnetic meridian, the horizontal directive force has no effect on the motion of the magnet: it is affected only by magnetic vertical force and by gravity.

The red end of the needle is pulled downwards, and the blue end is pushed upwards, by terrestrial vertical magnetism. To maintain the magnet in a horizontal position, its center of gravity cannot be below the knife-edge *C*, but must really be somewhere towards the blue end, as at *G*: the point *G* being

supposed to be connected with the magnet, and to vibrate with it. Let V be the magnet-power of earth's vertical force, B the magnet-power of the magnet; then as in Article 21, the angle θ being very approximately 90°,

$$V.B = W \times KG:$$

K being in the vertical below C, not vibrating with the magnet. It will be remembered that, in that Article, the units of statical and dynamical forces are connected by a formula which does not contain g.

Now suppose the Earth's vertical force V to vary. The only other element in the last equation which can vary is KG. Hence we find

$$B \times \delta V = W \times \delta . KG:$$

and, dividing this by the preceding equation,

$$\frac{\delta V}{V} = \frac{\delta . KG}{KG} .$$

But, as we cannot immediately measure $\delta . KG$, we must resort to an indirect process in order to extract a meaning from this equation. If the magnet is inclined through the small angle ψ, $\delta . KG$ will $= CK \times \psi$, and

$$\frac{\delta V}{V} = \frac{CK}{KG} \times \psi.$$

Now a value of CK may be obtained by causing the magnet to vibrate on its knife edges, thus. Let I be

the moment of inertia of the magnet: and consider the whole weight of the magnet, as assisted by the vertical magnet-force of the earth as above mentioned (the petty alteration of which will have no sensible effect on this element) to be collected at a point which, when the magnet is horizontal, coincides with K. Incline the magnet through a small angle χ: the angular moment produced by its weight will be

$$\frac{W.CK.\sin\chi}{I} \text{ or } \frac{W.CK}{I}\chi,$$

(omitting the usual factor g, for the reason given in Article 13): hence

$$\frac{d^2\chi}{dt^2} = -\frac{W.CK}{I}\chi.$$

The solution of this equation is,

$$\chi = K \times \sin\left\{t\sqrt{\frac{W.CK}{I}} + L\right\}.$$

Let T be the time of a complete double vibration, in which the variable term in the bracket increases by 2π; then $T\sqrt{\frac{W.CK}{I}} = 2\pi$, or $CK = \frac{4\pi^2.I}{W.T^2}$. Hence

$$\frac{\delta V}{V} = \frac{4\pi^2.I}{W.T^2}.\frac{\psi}{KG}.$$

To obtain a value for I, take the magnet off from its bearings, and suspend it by a single cord, as a free declination-magnet; the side which, when mounted on its bearings, is vertical, being now horizontal; so that

the same value I will now apply to its moment of inertia in horizontal vibration which formerly applied in vertical vibration. The magnet being now inclined to the meridian by the angle ω, and the force which acts on it being the horizontal magnetic force H, we shall have $\frac{HB \,.\, \sin \omega}{I}$ or $\frac{H.B.\omega}{I}$ for the angular moment: therefore

$$\frac{d^2\omega}{dt^2} = -\frac{H \,.\, B}{I}\,\omega:$$

$$\omega = M \,.\, \sin\left\{t\sqrt{\frac{H \,.\, B}{I}} + N\right\}:$$

and, if T' be the time occupied by a complete double vibration,

$$T'\sqrt{\frac{H \,.\, B}{I}} = 2\pi, \text{ or } I = \frac{T'^2 \,.\, H \,.\, B}{4\pi^2}.$$

Substituting this in the last expression

$$\frac{\delta V}{V} = \frac{T'^2}{T^2} \cdot \frac{H \,.\, B}{W \,.\, KG} \cdot \psi$$

$$= \frac{T'^2}{T^2} \cdot \frac{H \,.\, B}{V \,.\, B} \cdot \psi$$

$$= \frac{T'^2}{T^2} \text{ cotan dip} \times \psi.$$

This supposes that the unit by which δV is measured is the entire vertical force. If we prefer to adopt for unit the entire horizontal force, we have simply

$$\frac{\delta V}{H} = \frac{T'^2}{T^2} \times \psi.$$

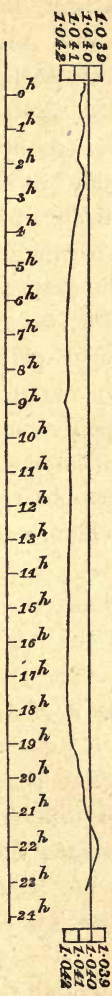
If (as with the other instruments) a concave mirror be attached to the magnet and throw the image of a fixed light to a photographic barrel (whose axis is vertical) at the distance p inches: then the direction of the reflected beam will be changed through the angle 2ψ, which will cause the light-spot to move through $2p \cdot \psi$ inches. For 1 inch of motion, ψ will $= \frac{1}{2p}$, and

$$\frac{\delta V}{H} = \frac{T'^2}{2p \cdot T^2}.$$

87. *Results obtained from the continuous registers of small changes in terrestrial magnetism.*

When the sheets of photographic paper are detached from their barrels, and a large number of these sheets (extending for instance through a year or through several years) are examined, they present the most capriciously discordant appearance that can be imagined. Thus, Figure 60 represents the curve given by the Horizontal-Force-Magnetometer on a quiet day (1869, October 17): Figure 61 represents that given by the same instrument on a day of disturbed magnetism (1869, March 10). It appears from such records that the terrestrial forces are at every moment in a state of change, though in very different degrees on different days. The laws of change extend without sensible alteration over considerable geographical distances: the writer of this treatise has compared many photographic records made at the Royal Observatory of Greenwich with those made at the same time

Fig. 60.


1.039
1.040
1.041
1.042
0h
1h
2h
3h
4h
5h
6h
7h
8h
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15h
16h
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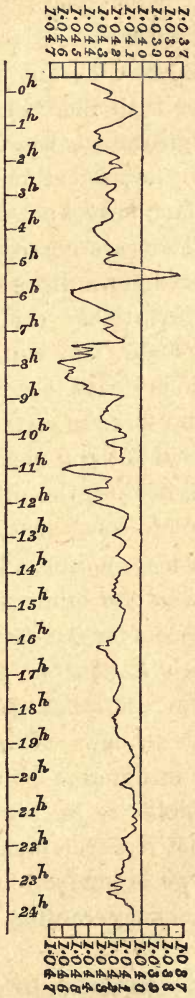
1.039
1.040
1.041
1.042

Fig. 61.


2.037
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2.043
2.044
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2.046
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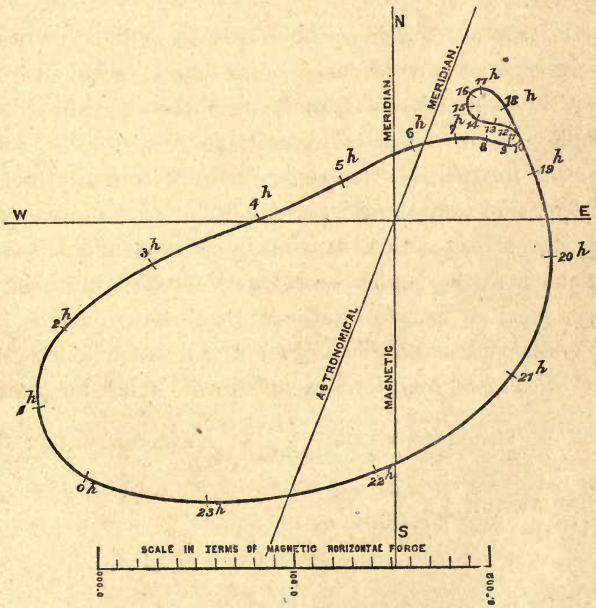
at the Kew Observatory, and has not remarked any sensible difference. In most cases, but not in all, the disturbances in the east or west direction are comparable with those in the north or south direction, and greater than those in the vertical direction. The periods of great disturbance sometimes occupy a portion of a single day, sometimes several days in succession: they are familiarly known by the name of 'magnetic storms.' They are not connected with thunder-storms or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of Aurora Borealis, and with spontaneous galvanic currents in the ordinary telegraph-wires: and this connection is found to be so certain that, upon remarking the display of one of the three classes of phænomena, we can at once assert that the other two are observable (the Aurora Borealis sometimes not visible here, but certainly visible in a more northern latitude).

But when the ordinates are picked out from the different sheets for the same hour of the day on every day through a year or through several years, the irregularities neutralize each other in a great degree: and the mean laws of inequality of the magnetic elements for different hours of the day have a very close resemblance, as deduced from different years. They are not however precisely the same: the change in their type is gradual, but it does not recur in any cycle of years or according to any other law yet established.

Having ascertained, from the mean of all the photo-

graphic records during a year, the mean value of the ordinate at each individual hour, and having compared that number for each hour with the mean of all the similar numbers for the 24 hours, we obtain the disturbance at each hour; in westerly force, or in northerly force, or in vertical force, as the case may be. Now if we lay down in a left-hand ordinate the westerly disturbance at each hour, and in an upright ordinate the northerly disturbance at each hour, we produce a curve of the singular form represented in Figure 62. The

Fig. 62



small figures on the curve give the solar hours. And

though there is a sensible difference (as has been stated) in the forms of the curves for different years, yet those characteristics of the curves upon which the eye rests as marking its most striking peculiarities are reproduced with accurate resemblance in all.

The curves for the different months have a marked difference. In the summer months, the curves are larger and more nearly round, and the small appendage about the early morning-hours (15^h, 16^h, &c.) is less strongly marked: in the winter, the curves generally are smaller, and the morning-appendage is more important.

If, instead of using solar hours to define the times of measure of our ordinates, lunar hours (reckoned from the time of moon's transit over the meridian) are employed, we obtain a remarkable result. The solar diurnal inequalities disappear entirely from the mean; and we find that there is a true lunar tide of magnetism, occurring twice in the lunar day, and shewing magnetic attraction backward and forward in the line from the Red Sea to Hudson's Bay. These forces are however considerably less than those which follow the law of solar hours. The mean diurnal solar inequality may be stated as about $\pm \frac{1}{600}$ of horizontal force: the lunar is about $\pm \frac{1}{12000}$.

$n = 20$

$\frac{1}{600} = n \frac{1}{12000}$

$n = 20.$

SECTION XII.

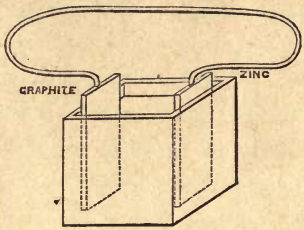
On the relation between Galvanic Currents and Magnetic Forces: and on the register of terrestrial galvanic currents: with special reference to disturbances of terrestrial magnetism.

88. *Fundamental principles of the creation of a galvanic current, and of its magnetic action: application to the galvanometer and to the speaking-telegraph.*

The subject of galvanic action in general belongs so completely to another science that we shall enter upon it here no farther than is absolutely necessary for explaining the relations of which this Section treats.

The simplest form of a galvanic battery is represented in Figure 63. A small vessel, of glass, or earthenware, or guttapercha, is nearly filled with dilute sulphuric acid. In the acid are plunged two plates of metal, selected principally for their difference of sus-

Fig. 63.

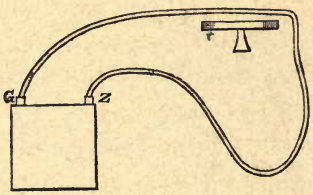


ceptibility to the action of the acid: great care is taken to prevent the plates from touching. For one metal, zinc (its surface usually being rubbed with quicksilver) has, from the beginning of the science, been adopted: for the other metal, silver, or copper, or (now more commonly) graphite, a form of carbon extracted from the iron retorts in which coal is distilled in the manufacture of gas for illumination. These two plates are connected by a metallic wire: or, a separate wire is soldered to each plate and the wires are brought into mechanical contact (which, if the touching surfaces are clean, is sufficient). Then a galvanic current or galvanic currents pass through the wire. We are justified in asserting this by observing that heat is produced in the wire, sometimes sufficient to fuse iron and platinum: that magnetic effects (to be mentioned shortly) are produced at every part of the wire: that time is required for the transmission of the effect through great length of wires: and that the disruption of the wire at any point produces a spark. The phænomena seem to justify us in asserting that a current proceeds from

each plate, the qualities of the two currents being different: and we shall sometimes, for convenience of language, speak of a "zinc current" and a "graphite current." Mechanically, the effects of such currents passing in the same direction may be considered as + and −: but when (as when originating from opposite ends of one wire) their directions are opposed, their effects are added together.

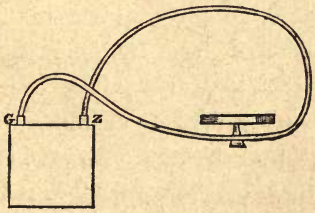
The simplest magnetic action produced by a galvanic current is the following. The current as in Figure 64 will deflect the red end of the needle from

Fig. 64.



the reader's eye. The current as in Figure 65 will

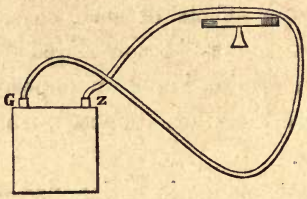
Fig. 65.



deflect the red end towards the reader's eye. The

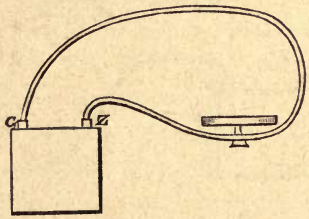
current as in Figure 66 will deflect the red end towards

Fig. 66.



the reader's eye. That in Figure 67 will deflect it from

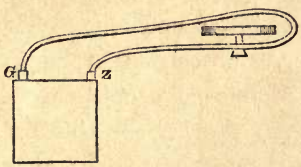
Fig. 67.



the reader's eye. The magnetic attraction is always normal to the direction of the current; a singular circumstance, we believe, in physical action. The direction may be remembered from the following fanciful rule. Conceive an insect to travel along the wire, in the direction of the graphite current, with his face always turned (upwards or downwards, or horizontally, as the case may be) to view the red end of the needle. Then the galvanic power deflects that red end towards his left hand.

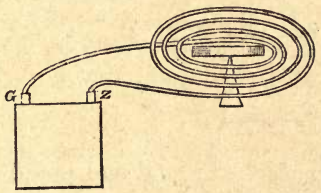
In Figure 68, the properties of Figures 64 and 67

Fig. 68.



are combined, and the red end is thrown from the reader with doubled energy. In Figure 69, the action

Fig. 69.



is multiplied to any extent. This is the construction of the ordinary galvanometer, and also that of the acting part in the common English speaking-telegraph.

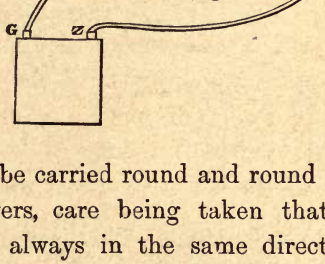
89. *Inductive magnetic power of the galvanic current: its action on steel and on iron; formation of transient magnets; registering-telegraphs.*

In treating of pure magnetism we have seen that a magnet-pole, which attracts the red end of a magnetized needle, possesses the power also, in the operation of double-touch, of drawing all the red magnetism of an

unmagnetized steel needle to the end nearest to itself, and thereby magnetizing that needle. And in like manner, if a soft iron bar be presented to it, it converts it for the moment into a magnet in a similar state. It is therefore easy to conceive that the galvanic current may be able to produce analogous effects.

The best form of wire for this purpose is a long spiral. In Figure 70 is exhibited a simple spiral: but

Fig. 70.

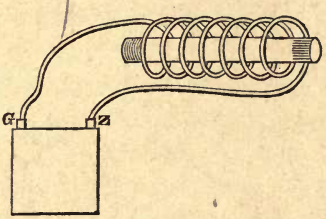


the wire may be carried round and round so as to form numerous layers, care being taken that the wire is turned round always in the same direction. Such a spiral constitutes a kind of magnet, which, though acting feebly on external objects, is sometimes useful. But its magnetic effect in the interior of the coil is powerful. Conceiving a mass of red magnetism in the interior, the imaginary insect which we have cited, in crawling along the wire from the graphite end, with his face towards the nearest part of the red mass, would in every part of the spiral have his left hand towards the graphite: and therefore the attraction of every part of the coil tends to draw the red magnetism towards

the graphite end, and the blue towards the zinc end. (If the direction of the spiral turns had been opposite, the result would have been opposite.)

Now if we insert in this coil a bar of unmagnetized steel, as in Figure 71, the bar is instantly magnetized,

Fig. 71.



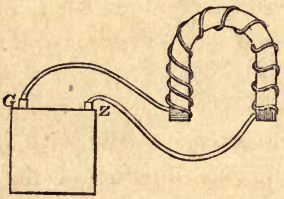
and becomes a magnetic needle with its red pole towards the graphite (the direction of the spiral being as shewn in the figure). This process is extensively used for magnetizing compass-needles.

If instead of the bar of steel we insert a bar of soft iron (usually called the 'core'), the bar is magnetized in the same manner as under other inductive force, having its poles in the same position as those of the steel bar just mentioned. But the magnetism is transient, lasting only as long as the galvanic current lasts. If the current be destroyed by interrupting the circuit in any way, as by cutting the wire at any point, or by separating two portions of the wire which are in contact, or by separating the wire either from the zinc plate or from the graphite plate, or by lifting either of the plates out of the acid,—in any of these cases, the iron instantly loses its magnetism. And this property

is exceedingly valuable, because by it we can make and unmake a magnet at a great distance, even several hundred miles, and in any locality, and even in a moving frame.

A convenient and powerful form is that of the horse-shoe magnet, the wires being arranged as in Figure 72. A piece of iron must be provided, to be

Fig. 72.



pulled by the two poles of the magnet. It is in this form that galvanism is commonly employed for the telegraphs in which permanent impressions are made on paper at the distant station.

90. *Spontaneous terrestrial galvanic currents: investigation of the magnetic effects due to them, and comparison of these magnetic effects with the magnetic disturbances recorded by the self-registering magnetometers.*

In the ordinary system of wire-telegraphs, each wire, when not used in the actual work of transmitting galvanic currents, is detached from all galvanic batteries, and is connected at both ends with the earth.

It was soon found that, when the wires are in this state, galvanic currents sometimes pass through them which are sufficiently strong to cause movement of the galvanometer-needle: and (when a battery is placed in the circuit for giving signals) sufficiently strong to pervert the telegraph-signals. And it was at length discovered that those currents, produced by the earth only, occurred at the same times as magnetic storms. In order to investigate the relation between the earth-currents and the magnetic storms, two wires were established in connexion with the Royal Observatory of Greenwich; one about 10 miles long, terminating at Croydon, the other about 8 miles long, to Dartford: each wire was carefully insulated in every part except at both its extremities, which were plunged in earth, and the two wires passed through two galvanometers, one appropriated to each wire, in the Observatory. Each of these wires might be expected to bring from the earth at one end to the earth at the other end a portion of the galvanism which is flowing through the superficial strata of the earth.

As it was soon found that currents, weak or strong, were almost always perceptible in the movements of the galvanometer, a self-registering apparatus was prepared. To the needle of each galvanometer a small plane mirror was attached, and the light of a lamp shining upon the mirror was by lenses made to converge, to form a spot upon a revolving barrel covered with photographic paper. Thus two registers were obtained similar in general character to those of the

changes of Magnetic Elements, described in the last Section.

The ordinates of these curves (considered as measures of the terrestrial galvanic currents passing through their respective wires) were measured for corresponding times. In order to determine experimentally the sign to be given to a current, considered as positive when its nature was that of a graphite current coming from the distant station, a small battery was placed so as to send graphite currents through the galvanometer, and the nature of the movements produced by it was noticed. Then it was conceived that each current might be represented as the effect of one current from the north and one from the west, the effect of each upon one experimental wire being proportional to the cosine of the angle made by that experimental wire with the north and with the west respectively. Putting α for the azimuth of Croydon from magnetic north towards east, α' for that of Dartford, C, D, N, W, the currents from Croydon, from Dartford, to the North, and to the West, respectively:

$$C = -N \times \cos\alpha + W \times \sin\alpha,$$

$$D = -N \times \cos\alpha' + W \times \sin\alpha';$$

from which

$$W = C\frac{\cos\alpha'}{\sin(\alpha - \alpha')} - D\frac{\cos\alpha}{\sin(\alpha - \alpha')},$$

$$N = C\frac{\sin\alpha'}{\sin(\alpha - \alpha')} - D\frac{\sin\alpha}{\sin(\alpha - \alpha')};$$

and by means of these expressions, the intensities of the northerly and westerly currents could be computed for every instant. Then, assuming these to be the representations of veritable currents flowing through the upper strata but below the Magnetic Observatory, and applying the rule of Article 88, the disturbances of northerly magnetic force due to W were found, and the westerly magnetic disturbances due to N were found. These were compared with the actual disturbances of northerly magnetic force (or variations of H) and the actual westerly magnetic disturbances (proportional to disturbances of declination) registered by the Horizontal-Force-Magnetometer and the Declination-Magnetometer respectively. And the results were as follows:

(1) In the magnetic storms, the disturbances of magnetism in the horizontal plane are almost perfectly explained as the effect of the terrestrial galvanic currents.

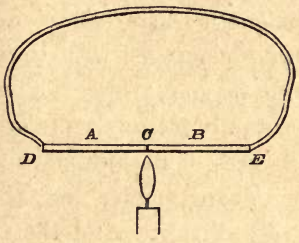
(2) On days of quiet magnetism, the magnetic forces, produced by the earth-currents, follow a well-marked diurnal law, which differs greatly from that of the magnetic diurnal irregularities.

(3) The galvanic currents discoverable at the earth's surface do not explain ordinary terrestrial magnetism. If that magnetism is to be explained by such currents, they must be very deep.

91. *Note on thermo-electric or thermo-galvanic currents, and on their possible connexion with terrestrial magnetism.*

In Figure 73, let A and B be two dissimilar metals,

Fig. 73.

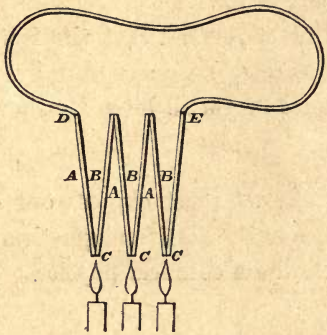


soldered together at C; and let their ends D and E be connected by a wire. (The metals found to be most favourable are antimony and bismuth.) Then if heat be applied at their point of union C, galvanic currents will be created through the wire DE, which can be measured by the deflexion of the needle of a galvanometer. The current which issues from the antimony-end of the combination is of the same quality as that which issues from the graphite-end of a galvanic battery.

If there be a number of pairs of bars of the same metals, as in Figure 74, and if heat be applied simultaneously to all the points of junction C, C, C, where the metals follow in the same order, the intensity of the galvanic current through DE is proportionally

increased. The several points *C*, *C*, *C*, are usually

Fig. 74.



brought very near together, in order to receive the same degree of heat.

This is the most delicate method known for measuring the intensity of radiant heat. It is thus that the diathermancy of different kinds of glass and of different crystals, &c., have been compared with great accuracy, and that the radiation of heat from the principal fixed stars has been made sensible.

Regarding the earth as a heterogeneous compound of different substances which may possess in some degree the properties of different metals, and conceiving (as is the opinion of many physicists) that there is in the interior a great store of caloric, which may heat the points of contact, some of them steadily and some by occasional bursts of flame, it seems within the

range of possibility that such a combination, of heat with dissimilar substances, may be the cause of terrestrial magnetism. But there is no evidence for this, beyond mere conjecture.

It is worthy of remark that the isothermal lines on the earth's surface bear a striking resemblance to the lines of equal magnetic intensity shown in Figures 35 and 36.

On the whole, we must express our opinion, that the general cause of the earth's magnetism still remains one of the mysteries of cosmical physics.

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